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Effective thermal conductivity of granular soils: a review of influencing factors and prediction models towards an investigation framework through multiscale characters

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Abstract:

The effective thermal conductivity of soil is important to geo-engineering applications, and it is controlled by factors across different length scales. Through a comprehensive review of these factors, we found that while other more traditional factors have been well studied, there is still a lack of characterisation of soil microscale and mesoscale structures and their influence on effective thermal conductivity. In addition, after reviewing the models available in the literature for soil effective thermal conductivity prediction, it was found that compared with empirical and theoretical models, machine learning models can account for the influence of multi-scale factors, however, research into them is scarce. To overcome the limitations of previous research, we proposed a framework that can investigate the factors influencing soil effective thermal conductivity at multiple scale. It includes the impact of soil structural factors at micro to mesoscale, and this impact is integrated with the influence from other factors for accurate thermal conductivity prediction.

Keywords
Soil thermal conductivity; Influencing factors; Prediction models; Soil fabric; Microstructures

1 Introduction

Heat transfer in geomaterials (soils and rocks) plays an essential role in the geotechnical and geology-engineering applications that contribute to a sustainable development, for example, hydrocarbon exploration (Schimmel et al. 2019), geothermal energy utilisation (Brandl 2006; Jia et al. 2019), thermal energy storage (Bauer et al. 2013), and carbon dioxide sequestration (Fei et al. 2015). Therefore, a clear and updated understanding of the heat transfer behaviour in geomaterials is of utmost importance to improve the reliability and productivity of associated engineering projects.

Soil is usually regarded as granular and composite material, and it mainly consists of solid particles and voids. The solid particles are made up of minerals or organic matters; and the voids are usually filled with water or air. Similarly, rocks can be thought as (mildly to highly) cemented granular materials, this is particularly true for sedimentary rocks. Consequently, there are three mechanisms driving heat transfer in soil: a) thermal conduction – heat is transferred from one solid particle to another if two particles contact each other (or through the solid cementation between them); b) thermal convection – the heat transference happens in the voids that contain water or air; and c) thermal radiation – heat transfer between different components and through electromagnetic waves at high temperature (Asakuma et al. 2014). Thermal conductivity (λ) is the property that indicates materials ability to transfer heat, and the λ of different soil constituents varies. In particular, the λ of solid particles is of different magnitude compared with that of water or air in the voids (Yun and Santamarina 2008). For example, $\lambda_{minerals}$ > 3 W/(m·K), λ_{water} = 0.56 W/(m·K), λ_{air} = 0.026 W/(m·K) at room temperature and atmospheric pressure. Considering the three heat transfer mechanisms in soil and the variation of λ between different components, the *effective* thermal conductivity λ_{eff} is adopted to indicate the overall heat transfer ability. Therefore, studying and predicting λ_{eff} is crucial in understanding soil heat transfer behaviour.

The soil λ_{eff} is controlled by soil structure, which can be quantified at different scales: macroscale, mesoscale, and microscale. *Macroscale* structures are derived from regarding different phases in soil as a corresponding whole unit, while ignoring, for example, the connection between solid particles and particles shape and size. For instance, porosity is a macroscale parameter, and it is defined as the ratio of the volume of voids to the total volume of soil. Porosity is mostly used to predict the λ_{eff} because it controls the contribution of different heat transfer mechanisms (e.g., heat conduction or heat convection) to overall λ_{eff} (Yun and Santamarina 2008; Côté and Konrad 2005; Rizvi et al. 2020c). A number of studies have investigated the soil λ_{eff} based on macroscale factors and corresponding models have been proposed to predict the λ_{eff} (Zhang et al. 2017; Zhang and Wang 2017). *Mesoscale* structures involve different particles in soil and characterise the connectivity between them and/or their interrelations with the pore space. For example, particle connectivity, defined by network features based on complex network theory (Fei et al. 2019b), indicates thermal conduction skeleton in soil. *Microscale* structures focus on individual particles. They include information about particle size and shape, which control inter-particle contact area and are defined through parameters like roundness and sphericity (Hryciw et al. 2016; Lee et al. 2017; Fei et al. 2019a). Therefore, investigating the effect of factors at different scales on λ_{eff} is the key to comprehensive understanding of soil heat transfer behaviour. Nevertheless, effects of multiscale (from micro to mesoscale) structural parameters on λ_{eff} have not been comprehensively reviewed and summarised. Furthermore, emerging models (e.g., machine learning models) for soil λ_{eff} prediction have not been included in the previous review papers.

This article first reviews the relationship between soil λ_{eff} and various influencing factors at different scales with comprehensive supporting data from the literature. The factors that have not been fully researched yet are summarised. The soil λ_{eff} prediction models that involve different factors are assessed and their potential limitations are identified. To conclude, a research framework is proposed and demonstrated for investigating soil λ_{eff} through factors at multiple scales.

2 Influencing factors of effective thermal conductivity

For comprehensive review of all influencing factors, a detailed category including non-redundant factors is required, particularly when considering soil structure at different scales. A comparison between the categories in literature and the category adopted in this study is given. Based on the proposed category, the influence of each factor on λ_{eff} is then analysed.

2.1 Categories of influencing factors

2.1.1 Categories of influencing factors in literature

The factors influencing soil λ_{eff} have been categorised differently in the various studies. Table 1 summarises the methods in the literature to categorise those factors. The classification – compositional factors, environmental factors and other factors – was adopted in (Zhang and Wang 2017); however, this classification could be clearer if compositional factors are further divided into components and structures. Dong et al. (2015) divided the factors into soil constituent, soil type, water content and particle contact; but these categories were redundant, because soil constituent and soil type are correlated. Soil nature, soil structure and soil physical condition were three groups identified by Jin et al. (2017), whereas these groups were not independent either. Abu-Hamdeh (2003) broadly classified the factors influencing soil λ_{eff} into those inherent to soil itself and those can be managed or controlled; but this classification is too general to arrange all the specific influencing factors.

2.1.2 Categories of influencing factors in this study

To address the limitations of previous research in categorising the factors influencing soil λ_{eff} (Zhang and Wang 2017; Dong et al. 2015; Jin et al. 2017; Abu-Hamdeh 2003), this study classify those factors into three types: components properties, structures and environmental conditions, as presented at the bottom of Table 1. The "components properties" consist of the thermal conductivity of the solid material and that of the void typically filled with air or water. The "structures" are considered at three scale levels: macro-level, meso-level and micro-level. Macroscale factors describe the material as a whole, for example, porosity is regarded as a structural factor at macro-level. Mesoscale factors involve two or more particles, for example, particle connectivity belongs to the meso-level. The microscale structural factors, which are based on individual particles, comprise particle shape, particle size and interparticle contact area. In addition, environmental conditions are composed of temperature, density, pressure, and moisture content – these are all macroscale factors as well.

Table 1 Categories of the factors influencing soil effective thermal conductivity

References	**Categories**	**Factors**	**Limitations**
Zhang and Wang (2017)	Compositions	Mineral components; gradation; particle size and shape; interparticle physical contact, e.g., the number of contact points; change of soil structure during drying and wetting cycle;	Particle connectivity and particle contact area are not arranged properly
	Environmental conditions	Water content and movement; density; temperature;	
	Other	Properties of soil; ions, salts and additives; hysteresis	
Dong et al. (2015)	Constituent	Particle thermal conductivity; mineralogy	The categories are correlated: the soil constituent differences are due mainly to different soil type
	Soil type	Mineral; gradation; particle size and shape	
	Water content		
	Particle contact	Coordination number	
Jin et al. (2017)	Soil nature	Texture; mineralogy; particle shape and size	The categories are not independent: soil particle size and shape are classified into soil nature, but they impact the soil structures
	Structural condition	Porosity; particle arrangement	
	Physical condition	Water content; temperature; pressure	
Abu-Hamdeh (2003)	Factors inherent to soil itself	Mineralogy; composition;	The categories are too general to arrange all specific influencing factors
	Factors manageable	Water content; density; porosity	
This study	Components properties	Particle / air / water thermal conductivity; materials' elastic stiffness	
	Structures	Macro-level: porosity Meso-level: particle connectivity, e.g., coordination number, quantified soil skeleton (Fei and Narsilio 2020) Micro-level: particle size, particle shape, particle contact area	
	Environmental conditions	Temperature; pressure; moisture content	

2.2 Soil components properties influencing the effective thermal conductivity

Heat conduction within particles is an important heat transfer process in geomaterials. Therefore, particle thermal conductivity ($\lambda_{particle}$) influences the soil effective thermal conductivity (λ_{eff})

(Tarnawski et al. 2009). In general, high soil λ_{eff} could be resulted from high $\lambda_{particle}$ (Côté and Konrad 2005; Zhang et al. 2015b; He et al. 2020). The $\lambda_{particle}$ is determined by the minerals that compose particles, and it ranges from 1.8 to 8.8 W/(m·K) (He et al. 2020). Johansen (1977) summarised the thermal conductivity of the minerals at common ambient temperatures around 25 ℃, which shows quartz has the highest thermal conductivity around 7.7 W/(m·K), while mica and feldspar have the lowest values, both of which are around 2 W/(m·K). Other minerals, e.g., pyroxene, amphibole, olivine, and chlorite, possess a thermal conductivity ranging from 2 to 5.8 W/(m·K). Consequently, geomaterials and soils that mainly consist of quartz show a higher λ_{eff} than those composed of other minerals. There are many models considering $\lambda_{particle}$ for λ_{eff} prediction: series and parallel models by Wiener (1912), uniform model by De Vries and Van Wijk (1963), geo-mean model by Johansen (1977) and Hashin and Shtrikman boundary model (Hashin and Shtrikman 1962; Yun and Santamarina 2008). While ignoring the influence of particle connection and contact on λ_{eff}, they could be used to show the relationship between λ_{eff} and $\lambda_{particle}$ to some extent. The equation based on geo-mean model (Johansen 1977) is selected for the showcase considering the available data from He et al. (2020).

$$\lambda_{eff} = \lambda_{particle}^{1-n} \lambda_{water}^{S \times n} \lambda_{air}^{(1-S) \times n} \tag{1}$$

$$n = \frac{V_{void}}{V_{total}} \tag{2}$$

where $\lambda_{particle}$ is the particle thermal conductivity; λ_{water} is the water thermal conductivity; λ_{air} is the air thermal conductivity; n is porosity defined as the ratio of void volume V_{void} and total volume V_{total}; and S is degree of saturation (the ratio of water volume to void volume). Measured thermal conductivity data from Birch and Clark (1940) shows that λ_{eff} prediction error using this model is within 20% (Johansen 1977). Figure 1 exemplifies the influence of $\lambda_{particle}$ on soil λ_{eff} in dry conditions, assuming λ_{air} = 0.026 W/(m·K). The figure shows the $\lambda_{particle}$ affects λ_{eff} to a larger extent in soil with low porosity than in soil with relatively high porosity.

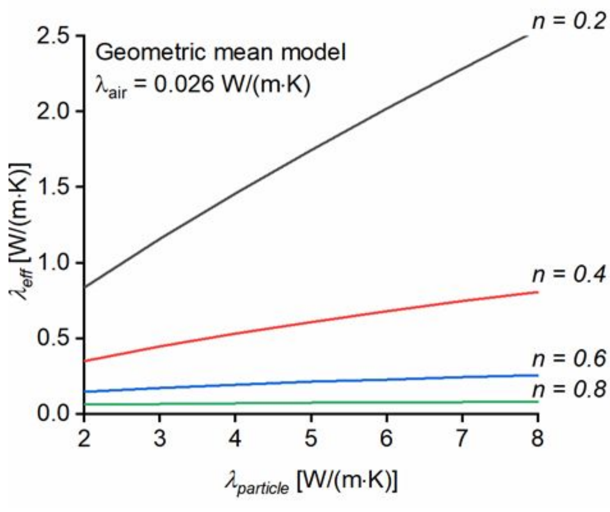

Figure 1 Influence of solid particle thermal conductivity $\lambda_{particle}$ on soil effective thermal conductivity λ_{eff} under different porosity based on data from He et al. (2020)

In addition to the solid particle thermal conductivity, elastic stiffness is another component property that affect the effective thermal conductivity (Morimoto et al. 2022). Morimoto et al. (2022) investigated the relationship between the thermal conductivity and granular materials' Young's modulus via the combination of DEM-generated granular samples and corresponding heat pipe network model. According to their findings, soil effective thermal conductivity may increase with its Young's modulus.

2.3 Soil structures influencing the effective thermal conductivity

2.3.1 Structural feature at macroscale

Porosity n is one of soil properties at macro-level and it refers to the fraction of pore volume V_{void} of total volume V_{total} as defined in Eq. (2) above. It is commonly used to describe the macro-structure of granular and sandy soils (Ding et al. 2023) and selected as a key/sole structural feature when establishing predictive models of thermal conductivity (Johansen 1977; Côté and Konrad 2005; Tong et al. 2009; Rizvi et al. 2020a). The void usually contains water and air. Since the values of particle thermal conductivity ($\lambda_{particle}$), water thermal conductivity (λ_{water}) and air thermal conductivity (λ_{air}) have different magnitudes and they together determine the soil effective thermal conductivity (λ_{eff}) to some extent, the porosity n influences the soil λ_{eff} remarkably .

Porosity is a result of soil texture and particle distribution. Sandy soil has a porosity range from 0.35 to 0.5, while finer soil porosity typically ranges from 0.4 to 0.6. Compacted soil possesses a porosity as low as 0.25 to 0.3 (Carter and Gregorich 2007). Overall, soil λ_{eff} decreases with the increase of porosity (Côté and Konrad 2005). Figure 2(a) shows the change of dry soil λ_{eff} with porosity. The included data are collected from Slusarchuk and Watson (1975); Johansen (1977); Côté and Konrad (2005); Yun and Santamarina (2008); Narsilio et al. (2010); Fei et al. (2019b). Soil λ_{eff} in dry conditions ranges from about 0.15 to 0.45 W/(m·K) while the porosity changes from about 0.25 to 0.55. Furthermore, Côté and Konrad (2005) claimed that dry soil λ_{eff} is directly related to porosity and they proposed the following exponential relationship between the two parameters:

$$\lambda_{eff,\,dry} = \chi \cdot 10^{-\eta n} \tag{3}$$

where $\lambda_{eff,\,dry}$ is the effective thermal conductivity of dry soil; χ and η are coefficients related to soil particle shape. They applied the equation to various soils: χ and η are 0.75 W/(m·K) and 1.2 for natural mineral sands, 1.7 W/(m·K) and 1.8 for crushed rocks, 0.3 W/(m·K) and 0.87 for organic fibrous soil. Figure 2(b) shows the change of unsaturated and saturated soil λ_{eff} with porosity. The experimental data for quartzite and granite are from Côté and Konrad (2005). It indicates that the porosity affects λ_{eff} to a larger extent with high degree of saturation, S, than low degree of saturation.

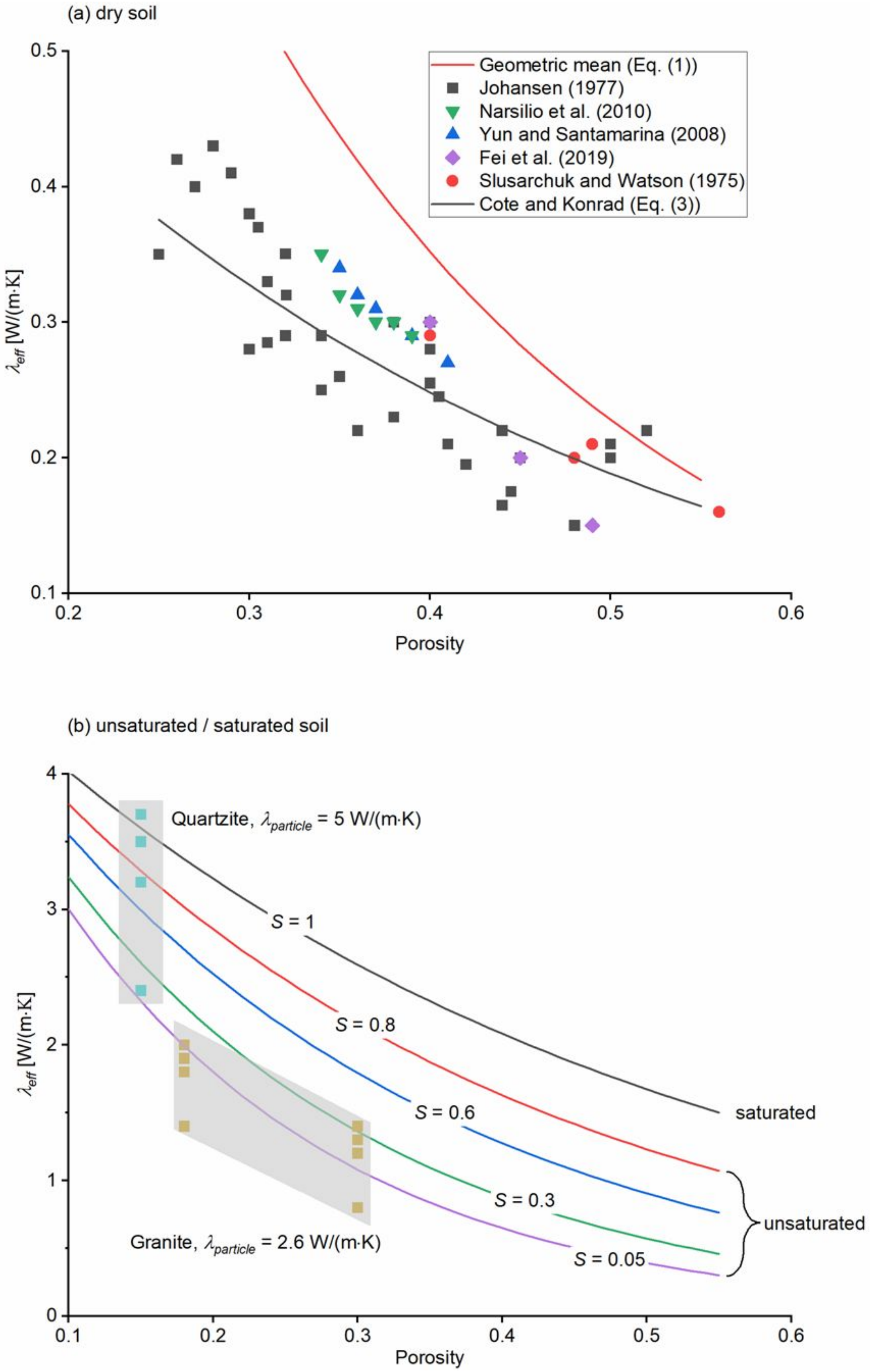

Figure 2 The distribution of soil effective thermal conductivity λ_{eff} with porosity: (a) dry soils with data from Slusarchuk and Watson (1975); Johansen (1977); Côté and Konrad (2005); Yun and Santamarina (2008); Narsilio et al. (2010); Fei et al. (2019b); χ and η are 0.75 W/(m·K) and 1.2 for generating Eq. (3); $\lambda_{particle}$, λ_{air}

and S are 2 W/(m·K), 0.026 W/(m·K) and 0 for generating Eq. (1); (b) unsaturated / saturated soils predicted values using Eq. (1) with $\lambda_{particle}$=5 W/(m·K), λ_{water}=0.56 W/(m·K), λ_{air}=0.026 W/(m·K); quartzite and granite experimental data is from Côté and Konrad (2005)

2.3.2 Structural features at mesoscale

Particle connectivity indicates soil skeleton and thus relates to the pathways of heat conduction between particles. Since heat conduction between particles dominates the heat transfer processes in soil, particle connectivity is important to soil effective thermal conductivity λ_{eff} (Dong et al. 2015). However, the indices of soil particle connectivity have not been well studied. Cheng et al. (1999) described the particle connectivity based on the results measured by Finney (1970), but only mono-sized spheres were considered. Particle connectivity could also be obtained if employing Discrete Element Modelling (DEM) for granular assemblies, since the contact between discrete particles and packing characteristics of granular systems can be found within DEM. Although research has been conducted on heat transfer using DEM, those work mainly focused on the computation of effective thermal conductivity and structural characterisation was out of their scope (Vargas and McCarthy 2001; Feng et al. 2009). Structural parameters such as packing fraction and macroscopic stress has been investigated by Peeketi et al. (2019) but further quantification of particle connectivity change due to altered packing and stress conditions is still missing. Many other studies used coordination number derived from DEM-generated granular assembly to investigate thermal conductivities (Yun and Evans 2010; El Shamy et al. 2013). Nonetheless, traditional coordination number alone cannot capture all information of complex granular materials' structure. For example, traditional coordination number only considers the number of contacts between particles but ignores the contact quality, which determine the thermal resistance between particles. Further description and quantification of particle connectivity cannot be achieved without other theories and tools.

Fei and Narsilio (2020); Fei et al. (2020) used complex network theory to characterise soil particle connectivity based on X-ray computed tomography (CT) images. To use complex network theory to characterise soil particle connectivity, a network representing soil particles has to be built first based on real soil CT images. This network's format is similar to that studied in (Feng et al. 2009; Yun and Evans 2010; El Shamy et al. 2013; Morimoto et al. 2022): soil is represented as a web composed of nodes and edges; the nodes represent individual particles, and edges exist between contacted particles. Then, the complex network theory can be applied to the built network to extract structural features. These structural features provide more comprehensive indices for particle connectivity and soil structure quantification, and these indices are classified into four types: centrality, network scale, cycles and clustering, as listed in Table 2. These different types describe structures of a soil particle assembly via different aspects (Newman 2003). The "centrality" type quantifies the importance of nodes (particles) within a network (granular assembly) based on their position or the number of paths passing through them. For example, the *degree* of a node in a network is the number of edges linked to this node, which is equivalent to coordination number in soil mechanics and powder technology. *Closeness centrality* of a node measures the how closely this node is related to other nodes, as exemplified in Figure 3 and detailed in Table 2. *Betweenness centrality* measures how often a node appears on the shortest path between two other nodes. The "network scale" measures the network size and interactions between nodes encompassed in a network. For example, *network diameter* is equal to the length of the longest one among the shortest paths between any two nodes in a network, as illustrated in Figure 3. The "cycle" type indicates the loop starting and ending at the same node and thus 3-cycles represent triangles. The "clustering" type measures the tendency of nodes to form tightly-knit groups or communities within a larger network as detailed in Table 2 and Figure 3.

Furthermore, weighted networks are established by adding contact area or thermal conductance as the weight to network edges, resulting in new weighted sub-indices under each type. In an unweighted network, an edge only indicates two nodes are connected. Even though the edges in Figure 3 have different lengths, the lengths have not been considered as weight for the edges. For network representing granular soils, if the contact area between two particles in physical contact is used to weight the edge between two nodes that represent those two particles in a network, the length of this edge has a physical meaning of the contact area. This process of adding weights to edges in a network bring physical meanings to the network features, and consequently the network features account for not only the number of particles contacts but also the contact quality. The influence from contact area / thermal conductance-weighted network features on heat transfer is also included in Table 2.

In general, soil λ_{eff} correlates directly proportional to the degree (also known as coordination number in soil mechanics and powder technology). For closeness centrality, soil λ_{eff} is related to the weighted value directly proportional while the unweighted value inversely proportional. Besides, soil λ_{eff} decreases with the increase of the betweenness centrality. In terms of network scale indices, soil λ_{eff} is inversely correlated to the average weighted shortest path. As for cycles and clustering, λ_{eff} increases with the number of 3-cycle in a network. Particularly, the weighted degree and weighted closeness centrality are identified as the best predictors for soil λ_{eff} (Fei and Narsilio 2020). A quantitative relationship between soil λ_{eff} and network features is given below (Fei et al. 2020) :

$$\frac{\lambda_{eff}}{\lambda_{solid}} = -0.21\left(\left[G^C\right]_{\kappa_w}\right)^2 + 0.67\left[G^C\right]_{\kappa_w} + 0.25 \qquad (4)$$

where λ_{solid} is the solid particle thermal conductivity; $\left[G^C\right]_{\kappa_w}$ is the *degree* weighted by the contact area between particles. Figure 4 illustrates the influence of *weighted degree* on soil λ_{eff} for different sands with data from Fei et al. (2021). The complex network methodology for soil structure quantification has not been extended to unsaturated soil, where water fills voids and bridges particles, and hence updated networks need to be proposed for studying heat transfer in unsaturated soil.

Table 2 Indices of soil particle connectivity at mesoscale based complex network theory and their relationship with soil effective thermal conductivity λ_{eff} (Fei and Narsilio 2020; Fei et al. 2020)

Type	**Sub-indices**	**Description**	**Correlation with λ_{eff}**
Centrality	Degree*	The number of edges linked to a node	Directly proportional
	Closeness centrality*	Related to distance of a node to other nodes and defined as $[G]_C(i) = \beta\left[\sum_{j=1}^{\lvert V\rvert - 1} d(i,j)\right]^{-1}$; $[G]_C(i)$ is closeness centrality of node i; β is a normalisation term; V is a node-set including i and j; $d(i,j)$ is the shortest path length from i to j.	Directly proportional for weighted values but inversely proportional for unweighted values
	Betweenness centrality*	Related to the extent that a node or edge connects other nodes or edges: $[G]_B(i) = \beta\sum_{j,k\in V}\frac{\sigma(j,k\mid i)}{\sigma(j,k)}$; $[G]_B(i)$ is betweenness centrality of node i; β is a normalisation term; $\sigma(j,k)$ is the number of $d(j,k)$; $\sigma(j,k\mid i)$ is number of $d(j,k)$ passing the node i.	Inversely proportional
	Eigenvector centrality*	The contribution of a node to network connectivity	
Network scale	Network diameter	The longest one among the shortest length paths: $[G]_D = Max_{i,j\epsilon V}[d(i,j)]$; $[G]_D$ is network diameter; V is the node set included in the network; $d(i,j)$ is the shortest path length from node i to node j.	
	Average shortest path length*	The average of the shortest length paths from every node to other nodes.	Inversely proportional
	Network density	The ratio of actual edge number to potential edge number	
Cycles	Number of 3-cycle	Number of loops in a network that starts and ends at the same node and has 3 edges	Directly proportional
Clustering	Global clustering coefficient	Indicating how integrated or fractured the network is: $G_{GC} = 3\frac{triangles'number}{connnected\ triples'number}$,	

Type	Sub-indices	Description	Correlation with λ_{eff}
		G_{GC} is the global clustering coefficient; a 'triangle' is a three nodes-set connected by three edges; a 'triple' is a three nodes-set connected by three / two edges .	
	Local clustering coefficient*	$[G]_{LC}(i) = \frac{2T(i)}{\kappa(i)[\kappa(i)-1]}$, $[G]_{LC}(i)$ is the local clustering coefficient of node i; $T(i)$ is the number of triangles passing node i; $\kappa(i)$ is the degree of node i.	

275 ** For the sub-indices calculated based on individual node or edge, the average value is adopted. The asterisk * indicates an average value.*

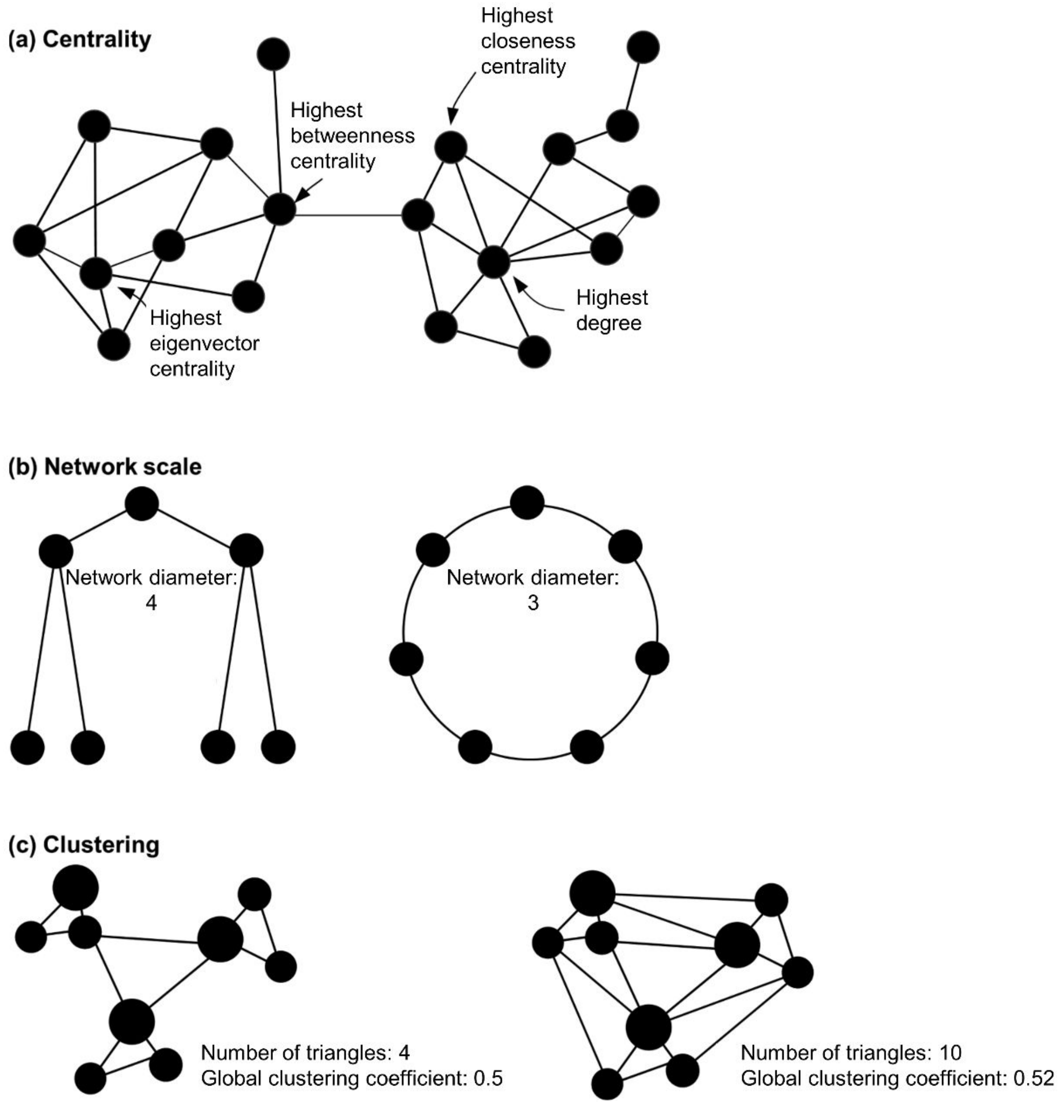

Figure 3 Network features illustration based on complex network theory from (Fei and Narsilio 2020)

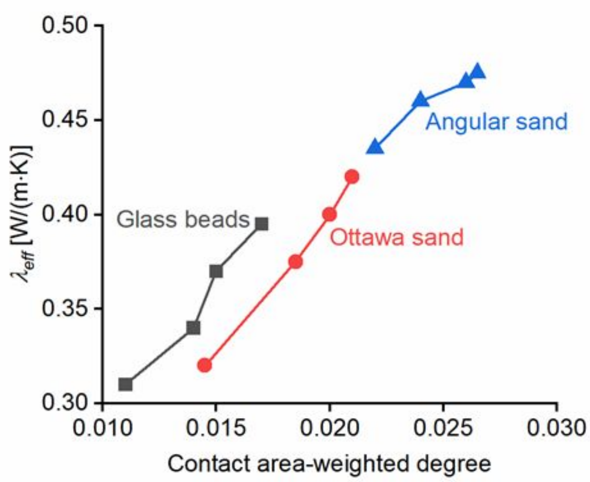

Figure 4 Influence of contact area-weighted degree in complex network theory on soil effective thermal conductivity λ_{eff} in with data from Fei et al. (2021)

2.3.3 Structural features at microscale

Soil particle shape and size control the contact area between particles, and the contact area governs the heat conduction process in soil. Since heat conduction between particles dominates the heat transfer processes in soil, particle shape and size are the key factors that influence soil effective thermal conductivity λ_{eff} (Gan et al. 2017; Lee et al. 2017; Fei et al. 2019a). Many studies adopt coefficients rather than particle shape and size descriptors themselves to consider the effect (De Vries 1963; Côté and Konrad 2005), where a quantitative relationship between λ_{eff} and particle shape and size is missing. To quantify the particle size and shape influence, this section firstly reviews the methods for particle size and shape description.

Two-dimensional microscopic images-based descriptors for particle shape, such as circularity, sphericity, roundness, etc., have been introduced in Cherkasova and Shan (2008); Cox and Budhu (2008); Lee et al. (2017) and (Xiao et al. 2020); however, they have some limitations when characterising irregular particles in natural soil, because the derived descriptor values may vary with the directions of projections (Fei et al. 2019a). Three-dimensional sphericity is introduced and used in Wadell (1932); Hamilton and Crosser (1962); Verma et al. (1991). Sphericity in these studies is defined as $\frac{Surface\ area\ of\ volume\ equivalent\ sphere}{Surface\ area\ of\ actual\ particle}$. Nevertheless, elongated particles cannot be well described using this definition. Lees (1964) described the angularity of particles using three main features: number of corners, corners degree of projection, and corners degree of acuteness. Zhou et al. (2018) combined roundness and sphericity to describe real sand particles with different shapes. Volume-based aspects ratio was found to have a positive correlation to thermal conductivity of silver nanofluids (Mirmohammadi et al. 2019). More recent works use sphericity in Eq. (5) and roundness in Eq. (6) for particle shape representation (Fei et al. 2019a). These two descriptors can be computed using CT images of real soil particles.

$$Sphericity = \frac{36\pi V^2}{(SA)^3} \tag{5}$$

$$Roundness = \frac{\sum \frac{r_i}{N}}{r_{\max,\,in}} \tag{6}$$

where V is particle volume and SA is particle surface area in Eq. (5); r_i is the radius of the i^{th} particle corner, N is the number of particle corners and $r_{\max,\,in}$ is radius of the maximum inscribed sphere in Eq. (6). Figure 5 shows the soil λ_{eff} change with different particle shape. In general, soil composed of spherical and round particles has a higher λ_{eff} than that composed of elongated and angular particles.

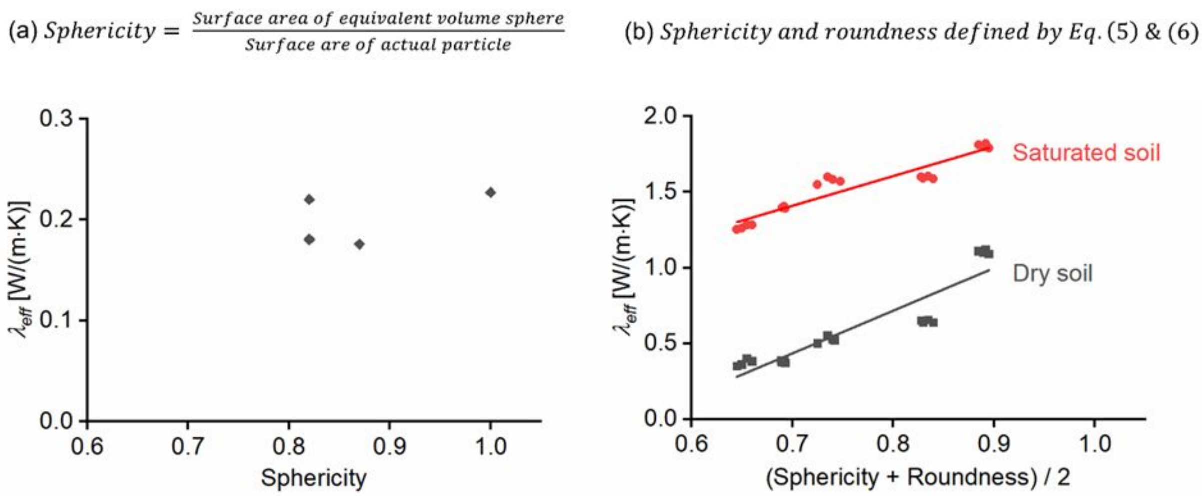

Figure 5 Influence of particle shape on soil effective thermal conductivity λ_{eff} (a) shape defined by surface areas with dry glass experimental data from Verma et al. (1991); λ_{solid} is around 1 $W/(m \cdot K)$, λ_{air} is around 0.025 $W/(m \cdot K)$; (b) shape defined by Eq. (5) and Eq. (6) with data from Fei et al. (2019a); $\lambda_{solid} = 3\ W/(m \cdot K)$, $\lambda_{air} = 0.025\ W/(m \cdot K)$, $\lambda_{water} = 0.591\ W/(m \cdot K)$

Parameters including 1) the fraction of relatively large particles in soil assembly and 2) particle mean or median diameters are usually used to study the influence of particle size on soil λ_{eff}. In most cases, soil with a larger particle size shows a higher λ_{eff} than that with smaller particle size (Zhang et al. 2015a; Gan et al. 2017). Gan et al. (2017) argued that small particle size leads to less particle contact area, and thus reduces the heat conduction between particles and lowers effective thermal conductivity in dry condition. It should be noted that, however, for unsaturated soil, small particle size leads to high soil λ_{eff} (Zhang et al. 2015b). This is because small particle size results in large surface area, and thus water films and bridges are easily formed between particles, thereby reducing thermal resistance considering that water has a greater ability for heat transfer than air. Furthermore, compared with the λ_{eff} of soil with low particle thermal conductivity, the λ_{eff} of soil with high particle thermal conductivity is more easily influenced by particle size (Zhou et al. 2010; Gan et al. 2017). Figure 6 presents general relationship between soil λ_{eff} and particle size. Mean particle diameters are used by Midttomme and Roaldset (1998), where particles are derived from synthetic samples with relatively uniform particle size. Median particle diameters are used by Fei et al. (2021), Lee et al. (2017) and Chen (2008), where diameters are from sieve analysis. The λ_{eff} relates more closely with mean diameters than median diameters.

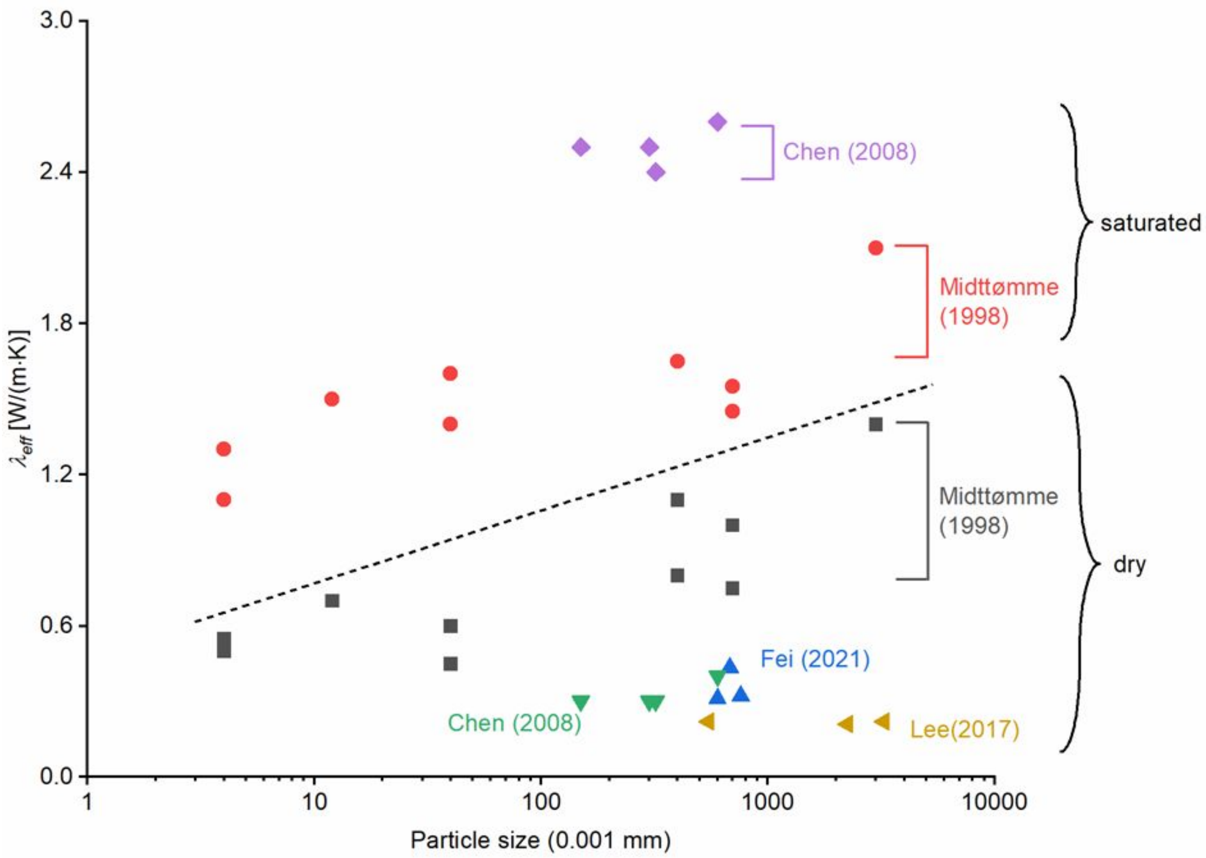

Figure 6 Influence of particle size on soil effective thermal conductivity λ_{eff} with data from Midttomme and Roaldset (1998); Chen (2008); Lee et al. (2017); Fei et al. (2021)

2.4 Environmental conditions influencing the effective thermal conductivity

2.4.1 Soil water content

The pores around solid particles in soil are typically filled with either air or water. Since water can reduce the thermal resistance between particles remarkably (Rao and Singh 1999), the water content, or more precisely the degree of saturation S (the ratio of water volume to voids volume) and volumetric moisture content θ (the ratio of water volume to soil volume) are considered as the prominent factors that affect soil effective thermal conductivity λ_{eff} (Zhang and Wang 2017; Agrawal et al. 2019; Lu and Dong 2015). Some studies report gravimetric moisture content w (the ratio of water mass to soil mass) rather than S and θ. In general, soil λ_{eff} under unsaturated or fully saturated conditions is higher than that under dry conditions (Johansen 1977; Chen 2008). Moreover, the soil λ_{eff} grows more rapidly at low S or θ compared with that at high S or θ (Zhang et al. 2015b; Dong et al. 2015). At low S or θ, solid particle connections are established gradually by water bridges formed in the voids. Considering thermal conductivity of water is greater than of air, these connections facilitate heat conduction in soil thereby increasing the soil λ_{eff}. In contrast, at high S or θ moisture content, solid particles are almost fully connected by water bridges so the soil λ_{eff} grows slightly. Moreover, the influence of water bridges depends on the void volume. Considering w is not as related to the void volume as S and θ, S and θ could be indicators that truly and generally influence λ_{eff}. Lu and Dong (2015) further introduced S_f and θ_f, at which the rate of change in thermal conductivity reaches maximum, for predicting soil λ_{eff}. Figure 7 shows the relationship between soil λ_{eff} and S with predicted values from De Vries (1963), Johansen (1977), Côté and Konrad (2005), Lu et al. (2007), Chen (2008), and experimental data from Zhang et al. (2015b).

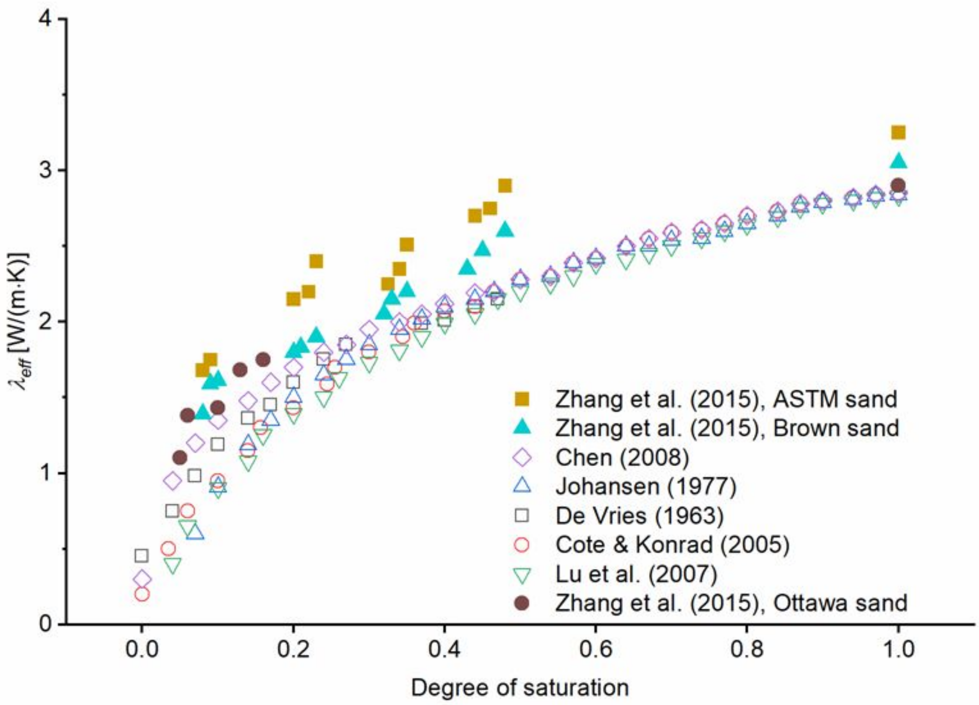

Figure 7 Influence of degree of saturation on soil effective thermal conductivity λ_{eff} with data from De Vries (1963); Johansen (1977); Côté and Konrad (2005); Lu et al. (2007); Chen (2008); Zhang et al. (2015b)

2.4.2 Soil temperature

Soil temperature influences the thermal conductivity of soil components, i.e., soil particles, air and water (Kayaci and Demir 2018). Besides, temperature can result in components phase change in soil and thus change the soil effective thermal conductivity (Gori and Corasaniti 2002). For example, under temperature below freezing point, part of fluid water in soil changes into solid ice, whose thermal conductivity is different from that of fluid water. The effects of temperature ranging from 30 ℃ to 90 ℃ are reviewed here. This temperature range is mostly encountered in shallow geothermal engineering, disposal of radioactive waste and thermal remediation of contaminated soils. In general, soil λ_{eff} rises with temperature, and its value at 90 ℃ is three to five times of that at ambient temperature (Campbell et al. 1994). This is attributed to latent heat transfer, caused by evaporation of water in soil voids, under high temperature and pressures different from atmospheric pressure (Liu et al. 2011). Furthermore, soil λ_{eff} increases more noticeably with temperature above 50 ℃ compared with that ranging from 30 to 50 ℃ (Smits et al. 2013), and temperature effect is negligible below 30 ℃ (Lu and Ren 2009). Besides, the impact of temperature on λ_{eff} is greater in moist soil than in dry soil, and it is most obvious when the soil degree of saturation is between 22% to 50% (Sepaskhah and Boersma 1979). Figure 8 visualizes the influence of temperature on soil λ_{eff} under different volumetric water content.

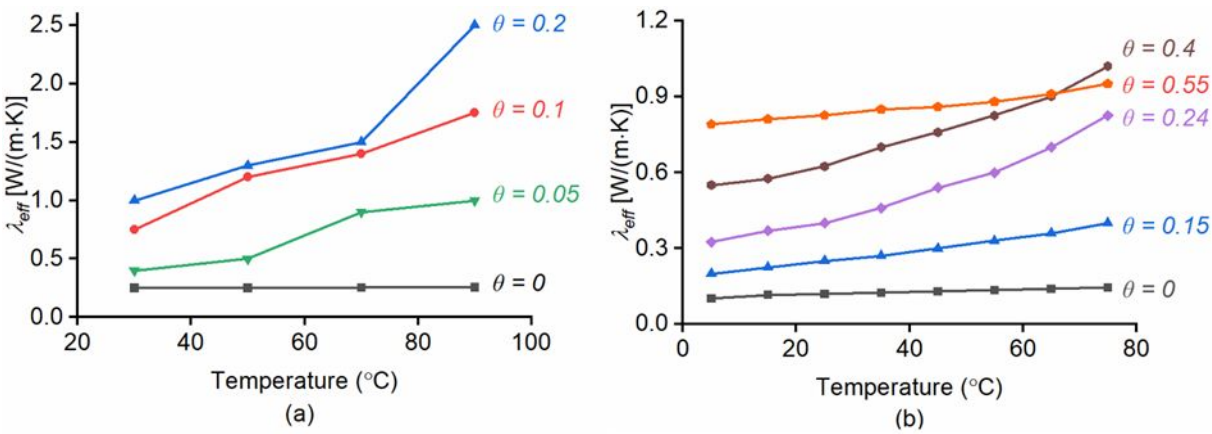

Figure 8 Influence of temperature on soil effective thermal conductivity λ_{eff} with experimental data selected from (a) quincy sand at different volumetric water content (Campbell et al. 1994) and (b) clay loam at different volumetric water content (Hiraiwa and Kasubuchi 2000)

2.4.3 Soil loading and gradation

Soil loading refers to the external forces acting on the soil beside self-weight. Soil loading induces compression thus increases the particle contact area between particles, as well as reducing porosity. Therefore, increasing loading increases the heat conduction process in soil and influences the soil effective thermal conductivity λ_{eff}. The λ_{eff} variation should also be attributed to the stress heterogeneity resulted from soil loading (Vargas and McCarthy 2001). Besides, soil structure (e.g., porosity, density, and particle connectivity) changes under loading, and consequently the soil λ_{eff} changes. In general, soil λ_{eff} is positively correlated to soil loading, and it almost increases linearly with the loadings (Vargas and McCarthy 2001; Weidenfeld et al. 2004). Weidenfeld et al. (2004) studied the effective thermal conductivity of particle beds composed of glass/limestone/aluminium etc. and found that λ_{eff} rises with different materials to different extents under compression. This finding should also be applied to soil considering the similarity between the particle beds and soil packings. Moreover, the influence of soil loading on λ_{eff} is negligible when the particle thermal conductivity is as low as less than 1 W/(m·K) (Weidenfeld et al. 2004). Compared with soil with small particle size, the λ_{eff} of that with large particle size is more easily influenced by the loading (Weidenfeld et al. 2004). Furthermore, the dependence of soil λ_{eff} on loading increases with the irregularity of particles because irregularity leads to more sensitive granular skeleton (Yun and Santamarina 2008). Cui et al. (2023) conducted a series of thermal test for soil specimens under loading-unloading conditions and at various degree of saturated. It was found that the change of thermal conductivity with loading-induced stress is more obvious under unsaturated condition compared with that under dry conditions. This effect is because that the addition of water improves soil suction and thus lowers soil compressibility. The influence of stress on λ_{eff} also depends on soil initial compression state: more loose soil tends to be more easily affected by stress. In addition, Xiao et al. (2018) studied the dependence of thermal conductivity on soil gradation, and it was found that thermal conductivity increases with soil uniformity coefficient.

3 Models for effective thermal conductivity prediction

After reviewing the influence of various factors on soil effective thermal conductivity λ_{eff}, models integrating those factors for λ_{eff} prediction are summarised in this section. Models for λ_{eff} prediction are mainly classified into three types: theoretical models, empirical models, and machine learning models.

Theoretical models are based on conceptual material geometry, and these models assume that different components in soil, i.e., solid, air, and water, are uniformly distributed. Then, the mathematical expressions for λ_{eff} are developed (Wiener 1912; De Vries and Van Wijk 1963; Gori 1983; Tong et al. 2009; Haigh 2012; Johansen 1977). Empirical models are proposed through comparing measured λ_{eff} with the value of different influencing factors (e.g., particle thermal conductivity, porosity, moist content). From this comparison, the key empirical coefficients that reveal the relationship between λ_{eff} and various factors can be drawn (Kersten 1949; Johansen 1977; Donazzi et al. 1979; Rao and Singh 1999; Balland and Arp 2005; Côté and Konrad 2005; Lu et al. 2007; Chen 2008). Machine learning models are based on trustable data and a learning process, which involve mathematic algorithms to establish the relationship between inputs (influencing factors) and outputs (λ_{eff}) (Grabarczyk and Furmański 2013; Li et al. 2022a). Table 3 summarises the considered factors and features of each model.

3.1 Theoretical models

Wiener (1912) defined the lowest and highest value of λ_{eff} by assuming that different phases in soil are ideally distributed. De Vries and Van Wijk (1963) model is a more complex one compared with Wiener model. It accounts for particle shape effect on λ_{eff} but the related coefficient is not easy to obtain. Johansen (1977) proposed a "geo-mean" model with a succinct mathematical expression. Gori (1983) model focuses on the λ_{eff} under different water distribution regimes and it is complicated to implement. Tong et al. (2009) model was developed from Wiener model. It is a comprehensive one because the effects of pore structure, degree of saturation and temperature are considered. Haigh (2012) model considers water film development (i.e., its width and thickness) when predicting λ_{eff}.

3.2 Empirical models

The λ_{eff} of soil with different temperatures, degree of saturation and mineral was measured by Kersten (1949), and he proposed two prediction equations for silts (or clay) and sandy soil respectively. In addition to the "geo-mean" model, Johansen (1977) also proposed "normalized thermal conductivity λ_r", which is expressed as:

$$\lambda_r = \frac{\lambda_{eff} - \lambda_{dry}}{\lambda_{sat} - \lambda_{dry}} \tag{7}$$

where λ_{dry} is thermal conductivity under dry conditions, and λ_{sat} is that under saturated conditions. Johansen developed several relationships between λ_r and degree of saturation S. And λ_r can be used to estimate λ_{eff} by interpolating λ_{sat} and λ_{dry}. This dimensionless coefficient has already involved many factors (e.g., soil type, minerology) and thus simplifies the prediction and widens the application range compared with the Kersten (1949) model. Balland and Arp (2005) model has an emphasis on the effect of organic matters on λ_{eff}. Côté and Konrad (2005) updated the λ_r- S relationship by considering soil type effect. Lu et al. (2007) claimed a linear correlation between λ_{eff} and porosity for dry soil. Chen

(2008) model has a good accuracy when predicting λ_{eff} of soil with high quartz contents. Other empirical models include Donazzi et al. (1979) model and Rao and Singh (1999) model.

3.3 Machine learning models

Machine learning models for λ_{eff} prediction are developed based on a large amount of trustable data regarding influencing factors and λ_{eff} (Wei et al. 2018). Typically, their architecture includes three layers: the input layer for influencing factors, the hidden layers for applying weights to the inputs, as well as the output layers for λ_{eff}. They can provide fast and convenient predictions when validated by trustable data.

Wei et al. (2018) used three methods: convolutional neural network (CNN), gaussian process regression (GPR) and support vector regression (SVR), to train available data and develop machine learning models. This work proves that machine learning models can provide accurate prediction. SVR and GPR are machine learning methods for non-linear regression analysis; and estimating porous media λ_{eff} from various factors is a non-linear problem. CNN has been widely applied in face recognition and thus it is able to capture the structure information in λ_{eff} prediction. Furthermore, six machine learning algorithms for soil λ_{eff} prediction are investigated in Li et al. (2022b). These algorithms include SVR, GPR, adaptive boosting method (AdaBoost), random forest (RF), decision tree (DT), and multivariance linear regression (MLR). The results show that AdaBoost provides good estimated values with the lowest error. Seven algorithms, including GPR, RF, DT, MLR, gradient boosting decision tree (GBDT), k-nearest neighbours (KNN), artificial neural network (ANN) were compared by Zhao et al. (2022) using different databases from Li et al. (2022b). These studies conclude that GPR, DT, and MLR are not the preferred algorithms for soil λ_{eff} prediction. Moreover, ANN was recommended by Zhao et al. (2022). A screen ANN is used to offset the influence of soil database insufficiency, and it utilises back-propagation algorithm in the training stage (Zhang et al. 2020). In addition, in order to balance the complexity with the accuracy of the prediction model, this study compares the model performances under different combinations of inputs. Rizvi et al. (2020a) developed an ANN model for unsaturated soil λ_{eff} prediction. Different parameters, including porosity, degree of saturation and quartz content, are used as inputs for the model; and the back-propagation algorithm is adopted for calculating the weight values in the ANN hidden layer. The same author also used a ANN based on group method of data handling (GMDH) to predict sand λ_{eff} (Rizvi et al. 2020b). Multilayer perceptron ANN is considered as the optimal one for the prediction of sandstone λ_{eff} (Vaferi et al. 2014). Mesoscale and microscale structures were firstly integrated into the inputs of ANN models in Fei et al. (2021). Inputs in his model consist of particle thermal conductivity, porosity, coordination number, particle roundness and sphericity. In general, machine learning models are able to account for more factors and can be applied to a wide range.

Table 3 Summary of prediction models for soil effective thermal conductivity λ_{eff}

Model category	**Author**	**Expression**	**Comments**	**Factors involved in each model**											
				λ, thermal conductivity, W/(m·K)			ϕ, volume fraction			n, porosity			S, saturation degree		
				ρ, dry density, kg/m^3			w, gravimetric moisture			T, temperature, ℃			C, coefficient		
				λ_{solid}	λ_{water}	λ_{air}	ϕ_{solid}	ϕ_{water}	ϕ_{air}	n	S	ρ	w	T	C
Theoretical models	Wiener (1912)	$\lambda_{eff,\,lower} = \left[\sum \frac{\phi_\alpha}{\lambda_\alpha}\right]^{-1}$ $\lambda_{eff,\,upper} = \sum \phi_\alpha \lambda_\alpha$ α indicates different phase	Determining the upper and lower boundary of λ_{eff}.	☑	☑	☑	☑	☑	☑						
	De Vries and Van Wijk (1963)	$\lambda_{eff} = \frac{\sum K_\alpha \phi_\alpha \lambda_\alpha}{\sum K_\alpha \phi_\alpha}$ K_α is the ratio of average thermal gradient of each constituent to that of continuous medium in soils	K_α is related to particle shape, and position, and it is difficult to be determined.	☑	☑	☑	☑	☑	☑						☑
	Johansen (1977)	$\lambda_{eff} = \lambda_{solid}^{1-n} \lambda_{water}^{S} \lambda_{air}^{(1-S)n}$	Developed from Wiener model (1912).	☑	☑	☑				☑	☑				
	Gori (1983)	Recommend referring to the literature. This model considers: 1) soil absorbed water content, 2) soil permanent wilting point, 3) soil field capacity.	Uncertainties exist in parameters used in this model.										☑		

Model category	**Author**	**Expression**	**Comments**	**Factors involved in each model**											
				λ, thermal conductivity, W/(m·K)			ϕ, volume fraction			n, porosity			S, saturation degree		
				ρ, dry density, kg/m³			w, gravimetric moisture			T, temperature, ℃			C, coefficient		
				λ_{solid}	λ_{water}	λ_{air}	ϕ_{solid}	ϕ_{water}	ϕ_{air}	n	S	ρ	w	T	C
	Tong et al. (2009)	$\lambda_{eff} =$ $\eta_1(1-n)\lambda_{solid} +$ $(1-\eta_2)$ $[1-\eta_1(1-n)]^2$ $\left[\frac{(1-n)(1-\eta_1)}{\lambda_{solid}} + \frac{nS}{\lambda_{water}} + \right.$ $+\eta_2$ $[(1-n)(1-\eta_1)\lambda_{solid}$	Coefficients η_1, η_2 relates to pore structure, saturation degree and temperature, which are difficult to be determined.	☑	☑	☑				☑	☑			☑	☑
	Haigh (2012)	Recommend referring to the literature. 1) Based on 2D soil contact cell unit, 2) Water film formation is considered, 3) Applicable for $n > 0.33$.	Involved coefficients are related to the thickness of water film, S and the width of water film, resulting in inconvenient implementation.	☑	☑	☑					☑				☑
Empirical models	Kersten (1949)	$\lambda_{eff} = 0.144[0.9 \log w - 0.2]10^{1.6\rho}$, for silts or clay $\lambda_{eff} = 0.144[0.7 \log w + 0.4] \times 10^{1.6\rho}$, for sandy soils	The applicable range of w is limited.									☑	☑		

Model category	Author	Expression	Comments	**Factors involved in each model**											
				λ, thermal conductivity, W/(m·K)			ϕ, volume fraction			n, porosity			S, saturation degree		
				ρ, dry density, kg/m^3			w, gravimetric moisture			T, temperature, ℃			C, coefficient		
				λ_{solid}	λ_{water}	λ_{air}	ϕ_{solid}	ϕ_{water}	ϕ_{air}	n	S	ρ	w	T	C
	Johansen (1977)	$\lambda_{eff} = \lambda_r(\lambda_{sat} - \lambda_{dry}) + \lambda_{dry}$ λ_r is the normalized thermal conductivity λ_{sat} is the saturated thermal conductivity λ_{dry} is the dry thermal conductivity	1) Proposing the concept of λ_r, 2) Obtaining λ_r by $\lambda_r \sim S$ correlation, 3) Obtaining λ_{dry}, λ_{dry} by his theoretical model.	☑	☑					☑	☑	☑			
	Donazzi et al. (1979)	$\lambda_{eff} = \lambda_{water}^{n}\lambda_{solid}^{1-n} \exp\left[-3.08n(1-S)^2\right]$	Easy to implement with no coefficients.	☑	☑					☑	☑				
	Rao and Singh (1999)	$\lambda_{eff} = 10^{1.6\rho}(1.07\log w + a)$ a is the coefficient	The coefficient is related to soils type. And $w \geq 10\%$ for clay and silts $w \geq 1\%$ for sands.									☑	☑		☑
	Balland and Arp (2005)	Recommend referring to the literature.	Predicting λ_{eff} through a proposed λ_r~S relationship.	☑		☑					☑	☑			☑

Model category	**Author**	**Expression**	**Comments**	**Factors involved in each model**											
				λ, thermal conductivity, W/(m·K)			ϕ, volume fraction			n, porosity			S, saturation degree		
				ρ, dry density, kg/m^3			w, gravimetric moisture			T, temperature, ℃			C, coefficient		
				λ_{solid}	λ_{water}	λ_{air}	ϕ_{solid}	ϕ_{water}	ϕ_{air}	n	S	ρ	w	T	C
	Côté and Konrad (2005)	$\lambda_{eff} = (\lambda_{water}^{n}\lambda_{solid}^{1-n} - a10^{-bn}$ $\left[\frac{cS}{1+(c-1)S}\right] + a$ 10^{-bn} a, b, c are empirical coefficients	Developing $\lambda_r \sim S$ correlation as $\lambda_r = \frac{cS}{1+(c-1)S}$.	☑	☑					☑	☑				☑
	Lu et al. (2007)	$\lambda_{eff} = [\lambda_{water}^{n}\lambda_{solid}^{1-n} - (b - an$ $\times$ $\exp\left[c(1 - S^{c-1.33})\right]$ $+(b - an)$, a, b, c are coefficients	The coefficients are related to the dry soil thermal conductivity and soil type.	☑	☑					☑	☑				☑
	Chen (2008)	$\lambda_{eff} = \lambda_{water}^{n}\lambda_{solid}^{1-n}$ $[(1-a)S + a]^{bn}$, a, b are coefficients	This model has high accuracy for high quartz content soil. The coefficients are related to soil type.	☑	☑					☑	☑				☑
Machine learning models	Rizvi et al. (2020a)	ANN algorithm	Only for unsaturated soil.				☑			☑	☑				
	Zhang et al. (2020)	ANN algorithm	Performances using different inputs are compared.				☑			☑	☑	☑			

Model category	**Author**	**Expression**	**Comments**	**Factors involved in each model**											
				λ, thermal conductivity, W/(m·K)			ϕ, volume fraction			n, porosity			S, saturation degree		
				ρ, dry density, kg/m^3			w, gravimetric moisture			T, temperature, ℃			C, coefficient		
				λ_{solid}	λ_{water}	λ_{air}	ϕ_{solid}	ϕ_{water}	ϕ_{air}	n	S	ρ	w	T	C
	Fei et al. (2021)	ANN algorithm	Mesoscale and microscale structure factors are considered.	☑						☑					
	Li et al. (2022b)	Six algorithms performance are compared	Adaptive boosting methods are recommended	☑			☑	☑		☑	☑		☑	☑	
	Zhao et al. (2022)	Seven algorithms performance are compared	Neural networks are recommended				☑			☑		☑	☑		

4 Research gaps and a methodological framework for futures studies

Based on the review regarding the factors influencing soil λ_{eff} and the models for the λ_{eff} prediction, a holistic view of research gaps is given, followed by detailed explanations. Then, a methodological framework for future studies is proposed.

In addition to the intrinsic properties of soil, its structure is an underlying factor that influences thermal conductivity, as it determines the structure of heat transfer pathways. Other factors, such as water content and gradation, also affect thermal conductivity by creating new structures for heat transfer pathways. Previous research has investigated the impact of soil structure on thermal conductivity in dry conditions; however, in unsaturated conditions, the addition of water connects soil particles, resulting in a different soil structure. Lu and McCartney (2024) and Lu and Dong (2015) have linked the different mechanisms of water retention to thermal conductivity. However, the altered heat transfer pathways due to the addition of water, which are underlying reasons contributing to an increase in thermal conductivity, remain an unexplored area of research. Furthermore, existing research on soil thermal conductivity has effectively utilised complex network theory to quantify soil structure, establishing correlations between network-derived features and thermal conductivity in dry conditions. This innovative approach marks a significant advancement in understanding soil behaviour. However, complex network theory alone may not fully capture the soil structure under unsaturated conditions where the structure undergoes notable changes due to water addition. In this light, the potential of other structural quantification methods should be explored. The alternative methods discussed below offer diverse perspectives on soil structure quantification, yet their parameters have not been investigated in relation to soil thermal conductivity.

Euler number is a topological invariant, and it is expressed as (Herring 2012; Herring et al. 2013; Herring et al. 2019)

$$\chi = \beta_0 - \beta_1 + \beta_2 \tag{8}$$

where χ is the Euler number; β_0 is the zeroth Betti number, representing the number of discrete elements in the volume; β_1 is the first Betti number, indicating the number of redundant loops in the structure; and β_2 is the second Betti number, referring to the number of cavities. Herring et al. (2013) used it to quantify the connectivity of nonwetting phase in porous media. However, the connectivity of soil pores that are based on Euler number has not been studied from the perspective of λ_{eff}. In addition, statistical approaches are also favourable to the description of soil structures. Minkowski functions are geometric measurements that can quantify the soil structure statistically based on computed tomography images of soil (Vogel et al. 2010). Specifically, the zeroth Minkowski function indicates total mass of the studied object (pore or solid); the first Minkowski function represents the interfacial area between pore and solid; the second is the interface's mean curvature; the third measures the total curvature (Vogel et al. 2010). The underlying theorem for using Minkowski functions to quantify the soil structure is proposed by Hadwiger (2013); he claimed that any properties, related only to the object's form, can be expressed by a combination of Minkowski functions. However, the relationship between Minkowski functions and soil λ_{eff} has not been researched. Furthermore, the particle or pore connectivity does not consider the local geometries (e.g., shape and size) of individual particles or pores; similarly, the local geometries (e.g., shape and size) do not include global information (e.g., particle or pore connectivity). But the soil λ_{eff} depends on both the global and local geometries. Persistent homology analysis can measure the global and local characteristic simultaneously (Herring et al. 2019). Therefore, parameters derived from persistent homology analysis could contribute to the comprehensive understanding of soil the λ_{eff}; whereas they have not been studied from heat transfer aspect. A parameter describing the extent of the transition from disorder to order in a granular system is proposed by Dai et al. (2019), which could also be introduced to soil structures quantification.

As previously pointed out, it is essential to quantify soil structure not only in dry conditions but also under unsaturated conditions to better understand the relationship between structural quantification and thermal conductivity. The goal of this endeavour is to include structural quantifications for thermal conductivity prediction. However, it is crucial to recognise that soil thermal conductivity is also influenced by other factors at various scales in addition to structural quantifications (Table 1). Therefore, a comprehensive framework that considers those additional factors presented in Table 1 is necessary for accurate prediction of thermal conductivity. Current models for predicting soil thermal conductivity fall short in this regard, as they do not fully account for all influencing factors, particularly the varied structure of unsaturated soils. The application of machine learning presents a promising avenue for developing a more integrative model (Fei et al. 2021). Current machine learning-based models in this field, however, have not yet fully incorporated soil structure data from both dry and unsaturated conditions as inputs. This limitation underscores the need for an updated machine learning framework that is designed to process and learn from a comprehensive set of inputs. By integrating detailed structural data from varying soil conditions along with other relevant factors at different scales, a framework is proposed in Figure 9, which could advance our capability to predict soil thermal conductivity with higher accuracy and relevance to real-world scenarios.

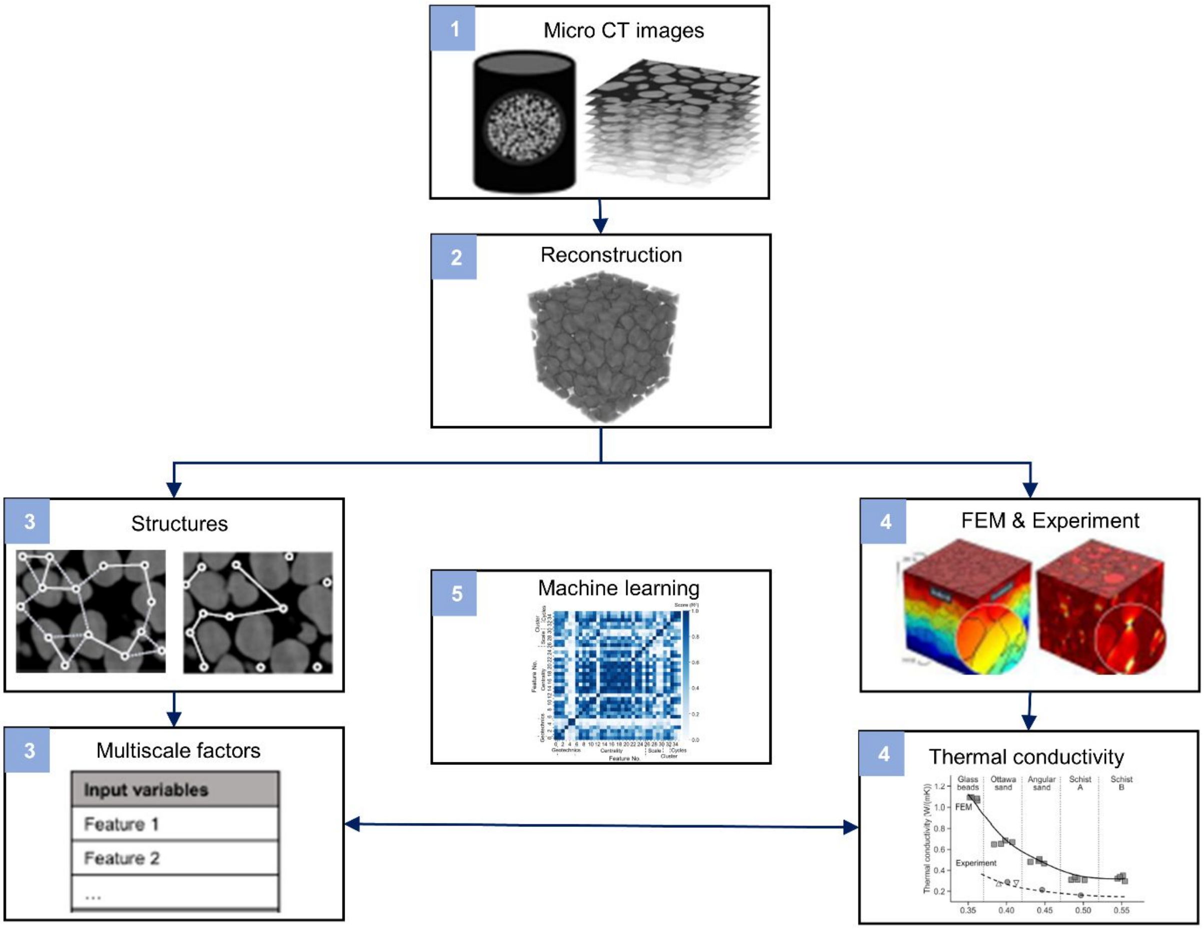

Figure 9 Framework of investigation on unsaturated soil effective thermal conductivity through multiscale characters; tools for each stage could be: 1) computed tomography imaging equipment; 2) software including ImageJ, Simpleware ScanIP; 3) network approaches or statistical methods; 4) finite element modelling and experimental measurements and 5) Python.

Firstly, modern computed tomography devices are employed to scan real soil samples (dry/unsaturated) and produce high resolution 3D image stacks. Afterwards, these images are used to reconstruct the 3D samples digitally by image process tools. The reconstructed 3D models serve as the foundation for both structural quantification and heat transfer processes modelling. The structural quantification relies on network approaches or statistical methods. Meanwhile, Finite Element Modelling (FEM) is adopted to simulate heat transfer processes to compute λ_{eff}, which will be further validated by measurements. Machine learning techniques are employed to discern the relationship between thermal conductivity and a combination of structural parameters and other traditional factors at multiple scales, including but not limited to solid particle thermal conductivity, porosity, and degree of saturation.

5 Conclusion

In this review, we systematically examined the various factors influencing soil thermal conductivity. Our findings highlight that soil structure impacts thermal conductivity significantly, but this area of research remains relatively unexplored due to the lack of characterising particle connectivity. A relationship between thermal conductivity and soil structure has been previously studied under dry conditions through the application of complex network theory for structural quantification. However, soil structure that undergoes notable changes due to the addition of water under unsaturated conditions has not been well characterised. Given the increased complexity of soil structure in unsaturated conditions compared to dry conditions, relying solely on complex network theory might be insufficient to capture the complete structural information. Consequently, we have explored other potential methods for a more comprehensive quantification of soil structure.

Furthermore, it is crucial to recognize that soil thermal conductivity is influenced not just by structural factors but also by a range of other variables. Our investigation reveals that current models for predicting soil thermal conductivity fall short of incorporating the full spectrum of influencing factors. To bridge this gap, we proposed a new integrative framework that considers both structural parameters and other relevant factors across different scales. This framework employs soil computed tomography (CT) images. These images offer a robust physical basis for an accurate description of soil structures based on quantification methods. Moreover, the framework integrates machine learning approaches, capitalising on their ability to assimilate a multitude of factors as inputs when predicting effective thermal conductivity. Machine learning's inherent strength in pattern recognition and data integration makes it particularly suited for this task. By combining the detailed structural data with other relevant factors, our framework aims to enhance the accuracy and applicability of predictive models, offering a more holistic understanding of soil thermal conductivity.

CrediT authorship contribution statement

Tairu Chen: Conceptualization, Data curation, Methodology, Formal analysis, Writing – original draft; Writing – review & editing., **Wenbin Fei:** Methodology, Writing – review & editing, Supervision., **Guillermo A. Narsilio:** Writing – review & editing, Supervision, Project administration, Funding acquisition.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Data Availability

Data generated or analysed during this study are available from the corresponding author upon reasonable request.

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