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11 **Impact of three-dimensional sphericity and roundness on coordination number**

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52 **ABSTRACT**

53 Coordination number (CN) is a fundamental micro-scale variable in soils affecting the macroscale
54 parameters of the material such as porosity, stiffness under loading and stability under hydraulic
55 gradients. However, most studies on CN have focused on sphere or ellipsoid packings using discrete
56 element method (DEM). By means of computed tomography (CT) and image processing techniques,
57 this work rigorously computes the three-dimensional sphericity and roundness of each grain in five
58 sands and investigates the impact of particle shape on CN. The results show that the average
59 coordination number (CN_{ave}) of *different* sands and the CN of grains *within a given* sand may be
60 impacted differently by particle shape. For a given equivalent diameter of a given sand, more irregular
61 grain packings show a subtle higher CN, this change in CN increases with increasing grain size.
62 However, air-pluviated irregular particle packings with poor gradation may exhibit lower average CN_{ave}
63 because of particle orientational alignment, particle interlocking and surface roughness.

64 **KEYWORDS:** particle-scale behaviour; fabric/structure of soils; sands

65

66 INTRODUCTION

67 Coordination number (CN) quantifies how many neighbouring particles surround a given particle in
68 a granular material. The *average* coordination number (CN_{ave}) of a soil affects the macroscale behaviour
69 and characterising parameters of the assembly. Shear-wave velocity in sandstones can be estimated
70 using CN_{ave} (Wang et al. 2019). The internal stability of a material under compaction or seepage force
71 can also be indicated by CN_{ave} . Triaxial compression tests revealed that higher shear stiffness was
72 attributed to higher CN_{ave} in sands (Fonseca et al. 2013). Hence, materials with high CN_{ave} can be used
73 to minimise particle breakage in pavements (Minh and Cheng 2013). The stability of an embankment
74 filter may decrease as CN_{ave} reduces monotonically in both sphere packings (Shire and O’Sullivan 2013)
75 and real sands (Fonseca et al. 2014). As CN_{ave} shows great importance, a study of how it is affected by
76 particle geometry (e.g. shape, gradation, friction, cohesiveness) is necessary because these geometrical
77 attributes may guide the material selection in engineering applications.

78 CN_{ave} generally increases with the particle size in non-uniform sphere packings (Minh and Cheng
79 2013). A better gradation can also increase the CN_{ave} in sphere packings (Minh and Cheng 2013). In the
80 discrete element method (DEM) study of packings of super-ellipsoids that can be represented by
81 Equation 1 (i.e., oblate and prolate spheroids), the correlation between CN_{ave} and aspect ratio (aspect
82 ratio is ‘ c ’ in Equation 1 after setting ‘ a ’ = ‘ b ’ = 1) shows an ‘M’ shape (Delaney and Cleary 2010;
83 Donev et al. 2004; Gan et al. 2017; Zhou et al. 2011) with the lowest CN_{ave} found at an aspect ratio of
84 1 (a sphere packing always has the lowest CN_{ave}). Additionally, the CN_{ave} reduces when ‘ m ’ shape
85 parameter ‘ m ’ changes from 3 to 5 (in Equation 1, increasing ‘ m ’ from 2 make a sphere more cubic-like
86 in shape) (Delaney and Cleary 2010) because of high particle orientational alignment:

$$\left(\frac{x}{a}\right)^m + \left(\frac{y}{b}\right)^m + \left(\frac{z}{c}\right)^m = 1 \quad (1)$$

87 where m is a shape parameter. a , b and c are principal axis lengths.

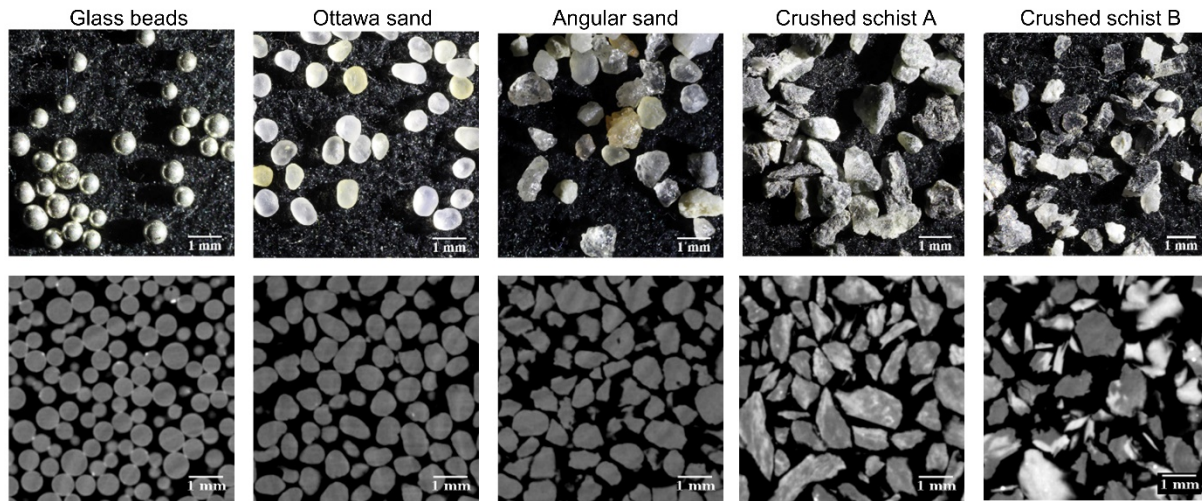
88 Similar results were found by Zhao et al. (2017) who used blockiness (defined as ‘ $1/m$ ’) to create
89 virtual particle assemblies in DEM. They also found that a high packing density does not necessarily
90 correspond to high CN_{ave} , cubic-like particle packings with high particle orientational alignment may
91 have high packing density but low CN_{ave} .

92 Even though DEM enables the effects of particle shape on CN_{ave} to be studied, the particle shape
93 indicators (aspect ratio) cannot cover the whole range of particle shapes encountered in natural sands
94 as discussed by [Fei et al. \(2019\)](#).

95 X-ray computed tomography (CT) sheds lights onto the analysis of microstructure ([Druckrey et al.](#)
96 [2017](#); [Fei et al. 2019](#); [Nadimi et al.](#)) in irregular natural sands and makes calculating three-dimensional
97 (3D) sphericity and roundness of each grain in the sands possible. The present work selected five sands
98 and rigorously quantified the particle size, 3D sphericity and roundness, CN of each grain in the sand
99 and the CN_{ave} of each sand from CT images. This work aims to bridge the gap between the particle
100 shape, CN of each grain and CN_{ave} in the real sands.

101 Materials

102 As shown in Fig. 1, five natural sands with distinct grain shape but similar diameter (D_{50}) were
103 selected. ‘Glass beads’ are nearly spheres made of silica with silver coats. ‘Ottawa sand’ ([ASTM 2017](#))
104 particles are more irregular than that of glass beads but still have relatively round corners. Similar to
105 Ottawa sand, grains in ‘angular sand’ are made of quartz but with sharp corners. ‘Crushed schist A’ has
106 even more irregular particles which are made of chlorites. Grains in ‘crushed schist B’ consist of both
107 quartz and biotite, have the most irregular shape and more than half of them are elongated and platy.
108 The D_{50} , the particle size range and the sorting index SI (the ratio of standard deviation to the equivalent
109 mean particle size, as defined by [Ahmed and Lebedev \(2019\)](#)) of these sands are summarised in Table
110 1. A soil with lower SI means that it is well sorted but poorly graded. Please note that these parameters
111 were calculated using four representative element volumes (REV) with dimensions of $4.5 \times 4.5 \times 4.5$
112 mm and were chosen from top-left, top-right, bottom-left and bottom-right regions of each sample. The
113 number of particles in a single REV excluding the particles on the border is also shown in Table 1.



114

115 **Fig. 1** Five sands with distinct particle shape. The pictures in the top row were photographed while the
 116 images in bottom row were CT scanned with a pixel size of 13 μm .

117 **Table 1** Particle size and sorting index (SI) of the studied granular materials

Sample	D_{50} (mm) *	D_{50} (mm) ^	Particle size range (mm) *	Particle size range (mm) ^	SI	Number of particles in a REV ^
Glass beads	0.60	0.60	0.50 – 0.70	0.40 - 0.80	0.14	270
Ottawa sand	0.73	0.76	0.60 – 0.85	0.58 - 0.94	0.22	118
Angular sand	0.89	0.68	0.60 – 1.18	0.39 – 0.99	0.22	100
Crushed schist rock A	0.84	0.58	0.50 – 1.18	0.23 – 0.95	0.36	119
Crushed schist rock B	0.84	0.61	0.50 – 1.18	0.16 - 1.10	0.28	94

118

* Diameter from sieve analysis.

119

^ Equivalent ball diameter calculated from particle volume after CT reconstruction.

120

121 Methods

122 The five sands are air-pluviated into relatively small PVC containers and scanned at the Australian
 123 Synchrotron to then extract particle shape and CN.

124 To date, various particle shape descriptors using different definitions have been proposed to
 125 characterise particles (Cho et al. 2006; Hryciw et al. 2016; Legland et al. 2016; Sneed and Folk 1958;
 126 Wadell 1932). Some two-dimensional (2D) descriptors were compared against 3D (true) descriptors in
 127 (Rorato et al. 2019). Upon a critical assessment and comparison of the existing definitions of particle
 128 shape descriptors, Fei et al. (2019) suggested that the combination of 3D sphericity (S in Equation 2)
 129 and roundness (R in Equation 3) can capture the shape of most irregular particles reasonably well:

$$S = \frac{36\pi V^2}{SA^3} \quad (2)$$

$$R = \frac{\sum r_i/N}{r_{max-in}} \quad (3)$$

130 where V is the particle volume, SA is the particle surface area, r_i is the radius of each one out of N
 131 corners in a particle and r_{max-in} is the radius of the maximum inscribed sphere of a particle. These
 132 variables and the CN of each grain in the packings are obtained following the procedures detailed in
 133 (Fei et al. 2019) with some corrections as outlined next.

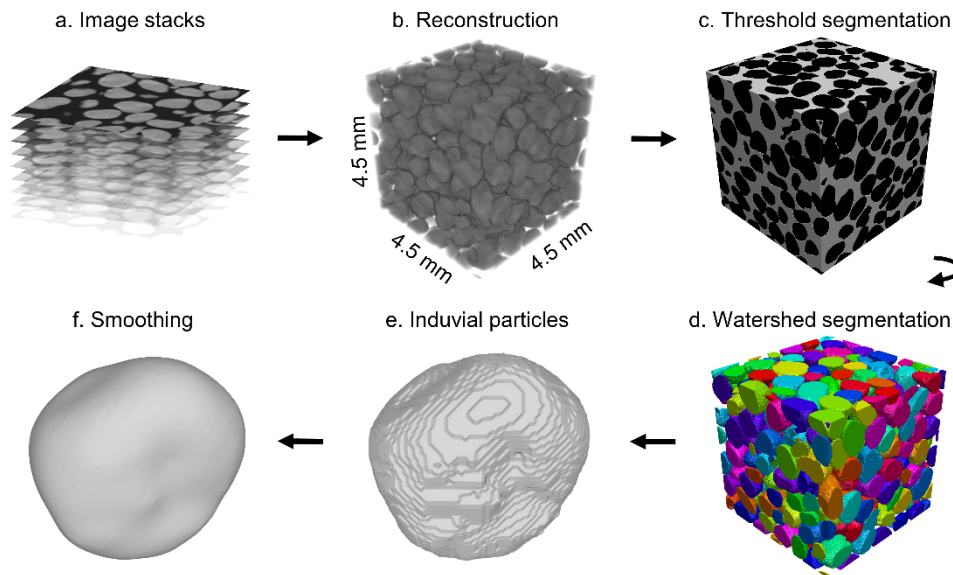
134 The isolation of individual grains from CT images is required to calculate their geometrical features.
 135 This is a non-trivial task given that grains in natural sands are in contact each other, thus the framework
 136 shown in Fig. 2 was proposed to reconstruct individual particles. Particle surface area SA and volume
 137 V are required to calculate sphericity S in Equation 2, whereas curvature $1/r_i$ in Equation 3 needs to be
 138 computed for roundness R . Pixel-counting with the saw-tooth particle surface (Fig. 2(e)) may result in
 139 overestimation of particle surface area and volume as well as incorrect roundness. Hence, the
 140 framework used here also includes a step to generate a smooth particle surface for more accurate
 141 estimations of sphericity and roundness from CT images.

142 Each sand was air-pluviated into a PVC cylinder ($d = 50$ mm, $h = 120$ mm) and scanned at the
 143 Australian Synchrotron to achieve sequential CT images with a pixel size of $13 \mu\text{m}$ (related to image
 144 resolution). The selection of sample size and pixel size is a trade-off, noting that images with a small
 145 pixel size (i.e., higher image resolution) can be used to identify more accurate interparticle contact area
 146 (Persson et al. 2004; Wiebicke et al. 2017) but does not significantly affect CN. CT images (Fig. 2(a))
 147 with a REV of $4.5 \times 4.5 \times 4.5$ mm (Fig. 2(b)) were firstly denoised by applying a median filter and a non-
 148 local mean filter in Fiji (Schindelin et al. 2012). These filters do not blur the images and keep the particle
 149 edge/boundary sharp. Applying the filters may comparatively smooth the particle surface but cause
 150 subtle loss of contacts formed by angular edges (Ahmed and Lebedev, 2019). The post-processed results
 151 were shown in Fig. 1. Next, the widely used Otsu threshold segmentation (Otsu 1979) was applied to
 152 binarize the solid and void phases (Fig. 2(c)). A plugin named MorphoLibJ (Legland et al. 2016) in Fiji
 153 (Schindelin et al. 2012) was employed to execute watershed segmentation on solids (Fig. 2(d)) and
 154 obtain individual voxelated particles (Fig. 2(e)). Six-voxel neighbourhood was selected in the watershed

155 segmentation to reduce the overestimation of particle surface area and volume (Fonseca et al. 2012). At
 156 last, the saw-tooth surface of the voxelated particles was smoothed by using the Taubin smoothing
 157 method (Taubin 1995). Taubin smoothing method includes two parameters λ and μ to control shrinkage
 158 and inflation, respectively. Here $\lambda = 0.6$ and $\mu = -0.3$ were determined after calibrating the particle
 159 surface area and volume of CT reconstructed sphere and ellipsoid against theoretical values, following
 160 (Fei et al. 2019).

161 An in-house program (Fei et al. 2019) was used to identify the interparticle contact by searching the
 162 shared boundary voxels between particles. The number of interparticle contacts was then used to
 163 calculate the CN of each grain and the CN_{ave} of each sample of the sands.

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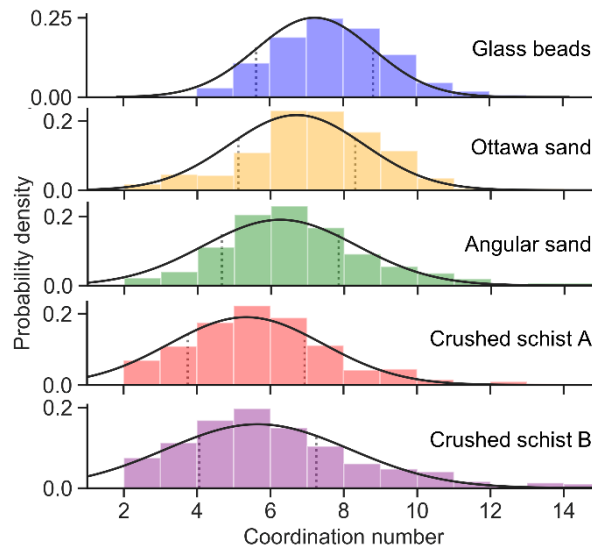
Fig. 2 Procedures to achieve individual particles of Ottawa sand from CT images

167 RESULTS AND DISCUSSION

168 For each sand, four subsamples from top-left, top-right, bottom-left and bottom-right regions of each
 169 sample were selected to calculate the CN and particle shape characteristics of every sand grain. The
 170 distributions of CN for the four subsamples from each material are shown in Fig. 3. It is clear from the
 171 fitted normal distribution curves that the CN_{ave} in the round glass beads is about 7. As glass bead size
 172 is not uniform, the maximum CN of a glass bead is 13 (slightly higher than in mono-dispersed Face
 173 Centred Cubic (FCC) sphere packings, which may indicate a slight overestimation of CN_{ave}). The CN_{ave}

174 then decreases to 5.3 and 5.7 in crushed schist A and B. The maximum CN in the very irregular crushed
 175 schist B is above 14 which is possible even for ellipsoids having strong friction (Donev et al. 2004).
 176 Besides particle shape, gradation also varies in all five sands according to the SI shown in Table 1.
 177 Hence, well-graded but more irregular particle packings may have lower CN_{ave} in real sands in contrast
 178 with the findings that better gradation can increase the CN_{ave} in sphere packings (Minh and Cheng
 179 2013). Fig. 3 shows that an irregular particle (e.g., crushed schist) may have higher CN than a round
 180 particle (e.g., Ottawa sand), this is true when one focuses on single grains. The effect of particle shape
 181 on average value of coordination number of a grain packing may be different from its effect on the
 182 coordination number of a single grain.

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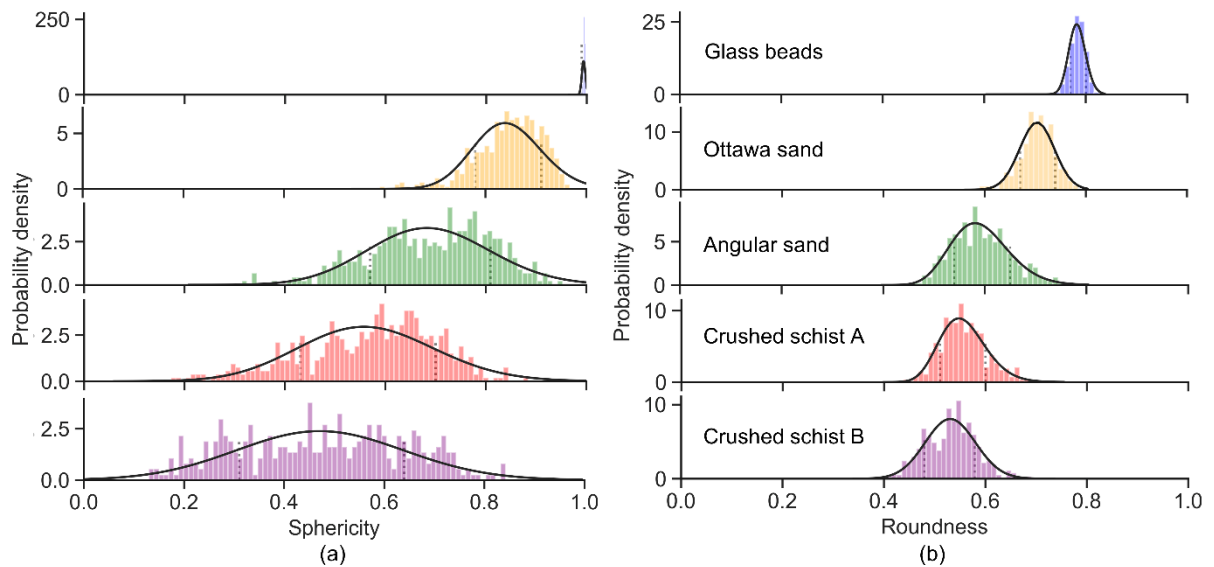


184

185 **Fig. 3 Distribution of coordination number and fitted normal distribution curve. The coordination**
 186 **number at the crest of the normal distribution curve is CN_{ave} . The vertical dotted lines show the limits of**
 187 **two standard deviations around CN_{ave} .**

188 To further quantify the impact of particle shape on coordination number, Fig. 4 shows the computed
 189 3D sphericity and roundness of each grain in the five sands and their probability distributions. Note that
 190 DEM studies typically use aspect ratio, a more straightforward parameter for ellipsoids than sphericity
 191 and roundness. Both sphericity and roundness range between 0 and 1. It is noticeable that the glass
 192 beads are not perfect spheres and the bright spots in the CT images of glass beads (Fig. 1) resulting
 193 from their silver coats could lead to the defects in reconstructed surface mesh, artificial low roundness

194 in some particles and larger CN_{ave} in the sample. The average sphericity and roundness decrease from
 195 round glass beads to irregular crushed schist B, which indicates that these descriptors capture well their
 196 particle shape. Excluding the glass beads, sphericity shows a larger range than roundness and the range
 197 of sphericity expands gradually from glass beads to crushed schist B.
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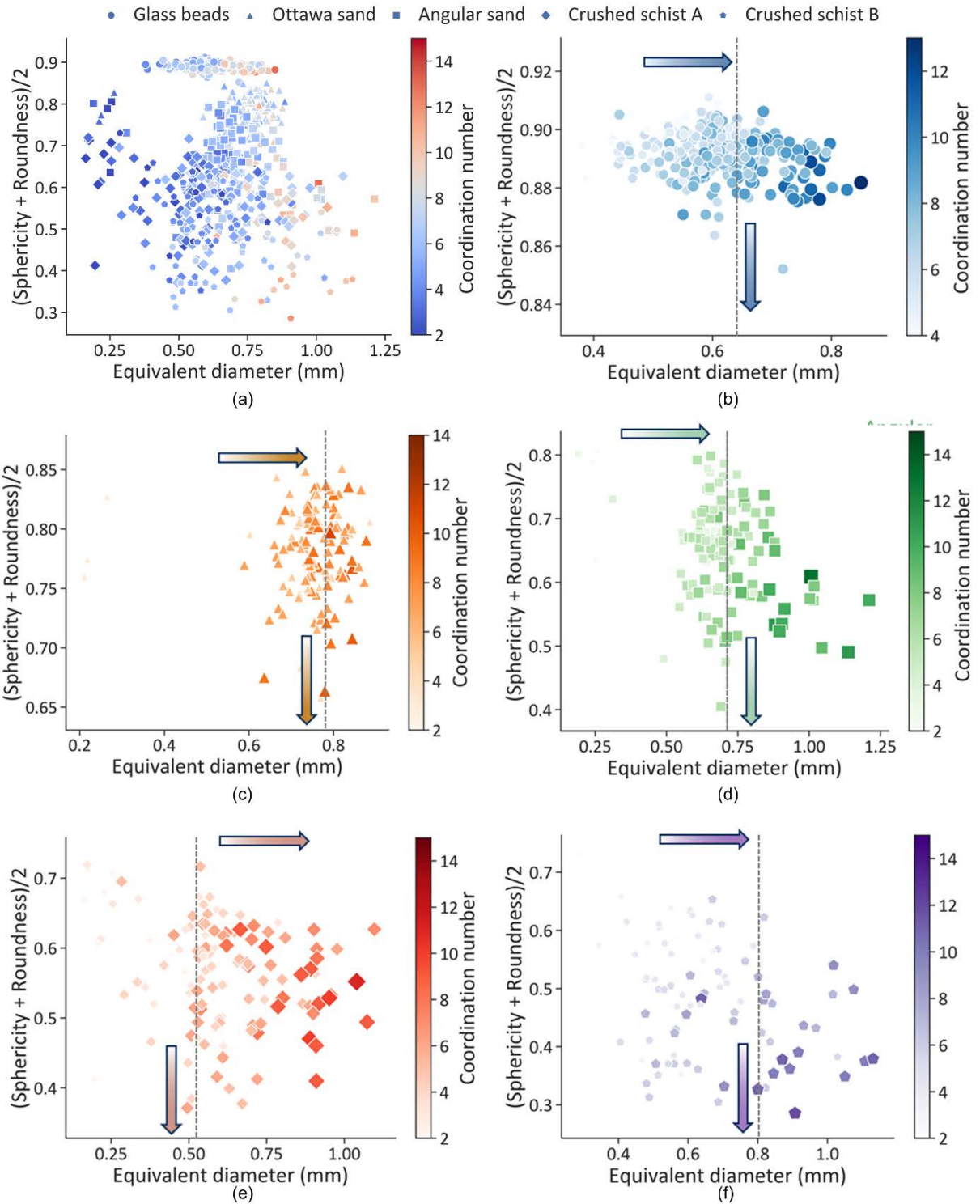


199
 200 **Fig. 4 Distribution of sphericity (a) and roundness (b) with fitted normal distribution curves. The**
 201 **vertical dot lines show the limits of two standard deviations around the average.**
 202

203 Since DEM studies set the ellipsoids having the same equivalent diameter, we also computed the
 204 equivalent diameter from the particle volume V in Equation 2. For a clearer visualisation, particles from
 205 only one of four REV of each specimen were used to show the relationship between equivalent
 206 diameter, the average of 3D sphericity and roundness, and CN of each grain in Fig. 5. The averaging of
 207 sphericity and roundness was proposed by [Cho et al. \(2006\)](#) based on the commonly used particle shape
 208 chart ([Krumbein and Sloss 1963](#)), and selected here since sphericity and roundness show similar trends
 209 to each other in Fig. 4. The red data points at the right of Fig. 5(a) shows larger CN because the grains
 210 have a larger equivalent diameter ([Fonseca et al. 2014](#)). This trend can also be clearly observed in each
 211 material from Fig. 5(b) – (f). Within a certain soil such as angular sand (Fig. 5(d)), the increasing trend
 212 is not only observed from left to right but also from top to bottom. This trend indicates that a more
 213 irregular large grain may exhibit higher CN in a given sand. For a given equivalent diameter, a subtle

214 increase in the CN of more irregular grains can also be seen along the dashed line in each sand, which
 215 is the same as the trend reported in DEM based papers (Delaney and Cleary 2010; Donev et al. 2004;
 216 Gan et al. 2017; Zhou et al. 2011) but different from the decreasing trend of CN_{ave} in more irregular
 217 sand packings as shown in Fig. 3.

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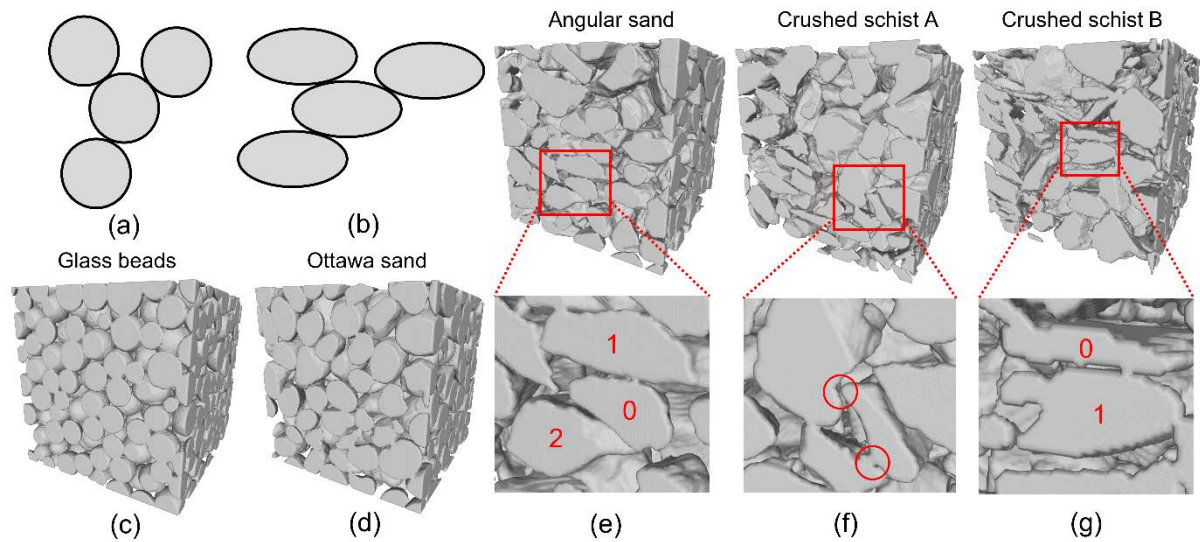
220 **Fig. 5 A subsample from each material are selected. For each grain, its particle shape is plotted**
221 **against equivalent diameter and rendered by coordination number. Grains from (a) combined**
222 **five sands, (b) glass beads, (c) Ottawa sand, (d) angular sand, (e) crushed schist A, (f) crushed**
223 **schist B.**

224

225 A qualitative explanation for these observations can be rooted in the micro-structural arrangement
226 of particles in a packing and the inherent (reasonable) assumptions used in DEM modelling. CN is
227 known to be related to the particle rolling and sliding when packing particles together under gravity
228 (Delaney and Cleary 2010; Donev et al. 2004). DEM studies show that ellipsoid packings have higher
229 CN_{ave} than that in sphere packings because an ellipsoid has more rotational degrees of freedom to be
230 restricted than a sphere in sphere packings (Delaney and Cleary 2010; Donev et al. 2004; Gan et al.
231 2017; Zhou et al. 2011) as shown in Fig. 6 (a) and (b). Let us illustrate this by focusing on the degree
232 of freedom on the two-dimensional planes in Fig. 6 (a) and (b): the central ‘spherical’ particle cannot
233 move when it is in contact with other three spherical particles. In contrast, an ‘ellipsoidal’ particle
234 restricted by three other ellipsoids can still rotate and have more neighbours to contact, resulting in a
235 higher CN for itself. 3D CT reconstructed geometry of each sand is also visualised in Fig. 6 (c) – (g) to
236 compare the particle arrangement. In the angular sand (Fig. 6 (e)), particle No.0 is jammed by only two
237 particles. In the more irregular crushed schist B (Fig. 6 (g)), the platy particle No. 0 can be supported
238 stably by a single particle No. 1. Hence, the orientational alignment of the irregular particles results in
239 a low CN_{ave} even though the single irregular particle has the possibility of high individual CN.
240 Additionally, the concave surface on irregular particles may induce interlocking so that a particle cannot
241 move even when touching fewer particles (Fig. 6 (f)). In addition, the irregular particle may also exhibit
242 higher roughness (another particle shape descriptor at a smaller scale than sphericity and roundness).
243 Higher roughness is related to a higher coefficient of friction which resists the rotation and sliding of
244 particles during gravity deposition and shear loading. However, DEM simulations usually set sliding
245 and rolling friction coefficient the same for all the particles in the assembly and also generally assumes
246 that particles with different particle shape have the same Young’s modulus (which may be different in
247 reality). Furthermore, the detection of CN in DEM is sensitive in the selection of a critical separation

248 distance (Zhou et al. 2011) which is also assumed as the same for all the particles. These limitations
 249 may be the cause of the contrasting observations made in this paper.

250



251

252 **Fig. 6 Schematic sphere packings (a) and ellipsoid assembly (b), and CT reconstructed glass beads (c),**
 253 **Ottawa sand (d), angular sand (e), crushed schist A (f) and crushed schist B (g).**

254 CONCLUSIONS

255 This study investigates the impact of particle shape on coordination number in real sands with the
 256 help of computed tomography (CT) and image processing techniques. In a given sand, a more irregular
 257 grain has subtle higher *individual* coordination number (CN) and the CN of the grain increases with
 258 increasing particle size. However, the trend may be different when comparing the *average* coordination
 259 number (CN_{ave}) in different sands. An air-pluviated more irregular particle packing with poor gradation
 260 may result in lower CN_{ave} due to particle orientational alignment, particle interlocking and surface
 261 roughness.

262 The trend of the CN_{ave} in this paper is different from that reported from DEM studies since this work
 263 is based on real sands and uses different particle shape descriptors. The reasons of this apparent
 264 discrepancy are a key finding in this work. It is hard to ensure that the different sands have the same
 265 average equivalent diameter as well as gradation compared with the flexible setting in DEM. However,
 266 DEM studies include simplifications such as setting a certain critical separation distance to detect all

267 particle contacts and usually assume that particles with different particle shape have the same Young's
268 modulus, sliding and rolling friction coefficients (they may be different in reality); all of which may
269 impact on the characterization of actual soils at the microscale, even in the absence of external load but
270 when using DEM to generate samples by gravity deposition.

271 The particle surface roughness and interlocking have not been quantified in this article and are
272 interesting topics for future research.

273 DATA AVAILABILITY

274 Some or all data, models, or code that support the findings of this study are available from the
275 corresponding author upon request.

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